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UNIVERSITY OF SWAZILAND

Supplementary Examination 2005

Title of Paper : Linear Algebra

Program : BSc./B.Ed./B.A.S.S. II

Course Number : M 220

Time Allowed : Three (3) Hours

Instructions :

1. This paper consists of SEVEN questions on THREE pages.
2. Answer any five (5) questions.
3. Non-programmable calculators may be used.

Special Requirements: None

THIS EXAMINATION PAPER MAY NOT BE OPENED UNTIL PERMISSION TO DO SO IS GRANTED BY THE INVIGILATOR.

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Question 1

(a) For what values of k is the following system consistent?

$$\begin{aligned} 2x + y &= -3 \\ x + 3y &= 2 + k \\ x - y &= 3k \end{aligned}$$

Solve the system in the consistent case(s).

[10 marks]

(b) Determine whether the subset $\{[2x,\ x+y,\ y] \mid x, y \in \Re\}$ is a subspace of $\Re^3$.

[5 marks]

(c) Let W_1 and W_2 be two subspaces of $\Re^n$. Prove that their intersection, $W_1 \cap W_2$, is also a subspace of $\Re^n$.

[5 marks]

Question 2

(a) Find a basis for the nullspace of the matrix: $\begin{bmatrix} 1 & 3 & 5 & 7 \\ 2 & 0 & 4 & 2 \\ 3 & 2 & 8 & 7 \end{bmatrix}$.

[10 marks]

(b) Find a matrix E such that: $E\begin{bmatrix} 1 & 3 & 1 & 4 \\ 0 & 1 & 2 & 1 \\ 3 & 4 & 5 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 3 & 1 & 4 \\ 0 & 1 & 2 & 1 \\ 0 & -5 & 2 & -11 \end{bmatrix}$.

[10 marks]

Question 3

(a) Using the set of linearly independent vectors in $\Re^4$, $\{[2,\ 1,\ 1,\ 1],\ [1,\ 0,\ 1,\ 1]\}$, form a basis for $\Re^4$.

[10 marks]

(b) Find a basis for the subspace of P, the space of all polynomials, spanned by $x^2 - 1,\ x^2 + 1,\ 4,\ 2x - 3$.

[10 marks]

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Question 4

(a) Find all real eigenvalues and corresponding eigenvectors of the matrix:

$$\begin{bmatrix} 2 & 0 & 0 \\ 1 & 1 & -2 \\ 1 & 0 & 1 \end{bmatrix}.$$

[10 marks]

(b) Find the unit vector in the same direction as $v = \begin{bmatrix} 3 \\ -4 \\ 12 \end{bmatrix}$. **[3 marks]**

(c) Determine whether the map $T': \Re^3 \to \Re^2$ defined by:

$$T([x_1, x_2, x_3]) = [x_1 + 2x_2 + x_3,\ x_1 - x_2 - 3x_3]$$

is a linear transformation. If it is, give its standard matrix representation.

[7 marks]

Question 5

(a) Find all solutions of the linear system, using the Gauss-Jordan method (transform the left partition of augmented matrix to reduced row-equivalent form).

$$\begin{cases} x_1 \quad - 2x_3 + \quad x_4 = 6 \\ 2x_1 - x_2 + x_3 - 3x_4 = 0 \\ 9x_1 - 3x_2 - x_3 - 7x_4 = 4 \end{cases}$$

[10 marks]

(b) Determine whether the set $\Re^2$ with the usual addition but with scalar multiplication defined by $r[x, y] = [ry, rx]$ is a vector space under these operations.

[10 marks]

Question 6

(a) Let u and v be vectors in $\Re^n$. Prove that $\{u, v\}$ is linearly dependent if and only if one of the vectors is a multiple of the other. **[5 marks]**

(b) Find a basis for the row space, a basis for the column space, a basis for the null space, the rank and nullity of the following matrix:

$$\begin{bmatrix} 2 & 3 & 1 \\ 5 & 2 & 1 \\ 1 & 7 & 2 \\ 6 & -2 & 0 \end{bmatrix}.$$

Hence verify the formula rank(A) + nullity(A) = n. **[15 marks]**

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Question 7

(a) Let $W = \text{sp}\{[1,\ 2,\ 1,\ 2],\ [2,1,\ 0,\ -1],\ [-1,\ 4,\ 3,8],[0,3,2,5]\}$ in $\Re^4$.
Construct a basis for W, and hence find $\dim(W)$.

[10 marks]

(b) Given the matrix $A = \begin{bmatrix} -1 & 2 & 1 \\ 2 & -3 & 5 \\ 1 & 0 & 12 \end{bmatrix}$, find A^{-1}, and hence write A as a product of elementary matrices.

[10 marks]

****** *END OF EXAMINATION* ******