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UNIVERSITY OF SWAZILAND

FINAL EXAMINATIONS 2005

B.Sc. / B.Ed. / B.A.S.S. II

TITLE OF PAPER	:	DYNAMICS I
COURSE NUMBER	:	M255
TIME ALLOWED	:	THREE (3) HOURS
INSTRUCTIONS	:	1. THIS PAPER CONSISTS OF SEVEN QUESTIONS. 2. ANSWER ANY FIVE QUESTIONS
SPECIAL REQUIREMENTS	:	NONE

THIS EXAMINATION PAPER SHOULD NOT BE OPENED UNTIL PERMISSION HAS BEEN GRANTED BY THE INVIGILATOR.

QUESTION 1

The position vector of a moving particle is given by

$$\mathbf{r} = 3\cos(2t)\hat{\mathbf{i}} + 3\sin(2t)\hat{\mathbf{j}} + (8t-4)\hat{\mathbf{k}}.$$

Find

(a) the velocity

(b) the speed

(c) the acceleration

(d) the magnitude of the acceleration

(e) the unit tangent vector

(f) the curvature

(g) the radius of curvature

(h) the unit principal normal

(i) the normal component of acceleration

(j) the unit binormal vector. [20]

QUESTION 2

(a) The force acting on a particle of mass m is given in terms of time t by

$$\mathbf{F} = a\cos\omega t\mathbf{i} + b\sin\omega t\mathbf{j}$$

If the particle is initially at rest at the origin, prove that the position at any later time is

$$\underline{\mathbf{r}} = \frac{a}{m\omega^2}(1-\cos\omega t)\mathbf{i} + \frac{b}{m\omega^2}(\omega t - \sin\omega t)\mathbf{j}$$

[6]

(b) A lift ascends 380 metres in 2 minutes travelling from rest to rest. For the first 30 seconds it travels with uniform acceleration, for the last 20 seconds with uniform retardation and for the rest of the time it travels with uniform speed. Calculate

(i) the uniform speed in metres per second; [4]

(ii) the uniform acceleration in metres per second squares [5]

(iii) the time taken by the lift to ascend the first 200 metres. [5]

QUESTION 3

(a) Given the three points $P(1,-1,2)$, $Q(2,-2,4)$ and $R(2,-1,3)$, find

(i) the angle between $\overrightarrow{PQ}$ and $\overrightarrow{PR}$ [2]

(ii) the area of the triangle whose vertices are given by the three points [2]

(iii) the equation of the plane passing through the three points. [4]

(b) Find the volume of the parallelepiped whose edges are the vectors

$$\mathbf{A} = 2\hat{\mathbf{i}} + 3\hat{\mathbf{j}} - \hat{\mathbf{k}}, \quad \mathbf{B} = \hat{\mathbf{i}} - 2\hat{\mathbf{j}} + 2\hat{\mathbf{k}}, \quad \mathbf{C} = 3\hat{\mathbf{i}} - \hat{\mathbf{j}} - 2\hat{\mathbf{k}}.$$

. [4]

(c) In cylindrical coordinates (r, θ, z), the position vector of an arbitrary point (x, y, z) is given by

$$\mathbf{R} = r\cos\theta\hat{\mathbf{i}} + r\sin\theta\hat{\mathbf{j}} + z\hat{\mathbf{k}}.$$

Show that, in this coordinate system the acceleration is given by

$$\mathbf{a} = \frac{d\mathbf{v}}{dt} = (\ddot{r} - r\dot{\theta}^2)\hat{\mathbf{r}} + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\hat{\theta} + \ddot{z}\hat{\mathbf{k}}.$$

[8]

QUESTION 4

(a) A particle of unit mass is thrown vertically upwards with initial speed V, and the air resistance at speed v is κv^2 per unit mass, where κ is a constant. Show that H, the maximum height reached, is given by

$$H = \frac{1}{2\kappa} \ln\left(\frac{g + \kappa V^2}{g}\right)$$

[10]

(b) Evaluate

$$\oint (2x - y + 4)dx + (5y + 3x - 6)dy$$

around the triangle in the xy-plane with vertices at (0, 0), (3, 0), (3,2) traversed in the counter-clockwise direction. [10]

QUESTION 5

(a) A particle of mass m moves in a central force field $\mathbf{F} = \frac{K}{r^n}\hat{\mathbf{r}}$ where K and n are constants. It starts from rest at $r = a$ and arrives at $r = 0$ with finite speed v_0. Prove that

$$v_0 = \left\{\frac{2Ka^{1-n}}{m(n-1)}\right\}^{1/2}$$

[8]

(b) Prove that

$$\nabla \cdot (\phi\mathbf{A}) = (\nabla\phi) \cdot A + \phi(\nabla \cdot \mathbf{A})$$

[7]

(c) If $\phi = x^2yz^3$ and $\mathbf{A} = xz\mathbf{i} - y^2\mathbf{j} + 2x^2y\mathbf{k}$, find

$$div(\phi\mathbf{A})$$

[5]

QUESTION 6

(a) Show, by means of the substitution $r = 1/u$, that the equation of a particle in a central field is

$$\frac{d^2u}{d\theta^2} + u = -\frac{f(1/u)}{mh^2u^2}.$$

[7]

(b) Suppose that a particle of mass m is acted upon by a force $\alpha r^{-2} + \beta r^{-3}$ per unit mass where $(\beta = \frac{1}{2}\alpha a)$ directed towards the origin $r = 0$ of an inertial frame. Suppose, also, that at $\theta = 0$ and $t = 0$, measurements of distance and velocity of the particle show that it is at distance a from the origin moving with velocity $\sqrt{\alpha/a}$ in a direction perpendicular to the radius vector. If $u = 1/r$, prove that

$$u = \frac{2}{a} - \frac{1}{a}\cos\frac{\theta}{\sqrt{2}}$$

[13]

QUESTION 7

(a) A car with initial speed u accelerates uniformly over a distance of $2s$ which it covers in time t_1. It is then stopped by being retarded uniformly to rest over a distance s, which it covers in time t_2. Prove that

$$\frac{u}{2s} = \frac{2}{t_1} - \frac{1}{t_2}.$$

[10]

(b) Two points A and B are at distance d apart. A particle starts from A and moves in the direction $\overrightarrow{AB}$ with initial velocity u and uniform acceleration a. A second particle starts at the same time from B and moves in the direction $\overrightarrow{BA}$ with initial velocity $2u$ and retardation a.

(i) Prove that the particles collide at time $\frac{d}{3u}$ from the beginning of the motion. [5]

(ii) Prove that if the particles collide before the second particle returns to B, then

$$ad < 12u^2.$$

[5]

END OF EXAMINATION