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UNIVERSITY OF SWAZILAND

SUPPLEMENTARY EXAMINATIONS 2005

B.Sc. / B.Ed. / B.A.S.S. II

TITLE OF PAPER	:	DYNAMICS I
COURSE NUMBER	:	M255
TIME ALLOWED	:	THREE (3) HOURS
INSTRUCTIONS	:	1. THIS PAPER CONSISTS OF SEVEN QUESTIONS. 2. ANSWER ANY FIVE QUESTIONS
SPECIAL REQUIREMENTS	:	NONE

THIS EXAMINATION PAPER SHOULD NOT BE OPENED UNTIL PERMISSION HAS BEEN GRANTED BY THE INVIGILATOR.

QUESTION 1

The position vector $\mathbf{r}$ of a moving point is given by

$$\mathbf{r} = (2t + 3)\hat{\mathbf{i}} + (t^2 - 1)\hat{\mathbf{j}}.$$

Find,

(a) the velocity vector $\mathbf{v}$ [1]

(b) the acceleration vector $\mathbf{a}$ [1]

(c) the speed [2]

(d) the unit tangent vector $\widehat{\mathbf{T}}$ [2]

(e) the curvature [5]

(f) the unit normal vector $\widehat{\mathbf{N}}$ [2]

(g) the unit binormal vector $\widehat{\mathbf{B}}$ [3]

(h) the tangential component of acceleration [2]

(i) the normal component of acceleration [2]

QUESTION 2

(a) In cylindrical coordinates (r, θ, z), the position vector of an arbitrary point (x, y, z) is given by

$$\mathbf{R} = r\cos\theta\hat{\mathbf{i}} + r\sin\theta\hat{\mathbf{j}} + z\hat{\mathbf{k}}.$$

Show that, in this coordinate system

(i) the velocity is given by

$$\underline{\mathbf{v}} = \frac{d\mathbf{R}}{dt} = \dot{r}\hat{\mathbf{r}} + r\dot{\theta}\hat{\theta} + \dot{z}\hat{\mathbf{k}},$$

[6]

(ii) the acceleration is given by

$$\underline{\mathbf{a}} = \frac{d\mathbf{v}}{dt} = (\ddot{r} - r\dot{\theta}^2)\hat{\mathbf{r}} + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\hat{\theta} + \ddot{z}\hat{\mathbf{k}}.$$

[6]

(b) Find the area of the triangle whose vertices are $A = (0,0,0)$, $B = (-2,3,0)$ and $C = (3,1,0)$. [4]

(c) For what values of a are $A = a\mathbf{i} - 2\mathbf{j} + \mathbf{k}$ and $B = 2a\mathbf{i} + a\mathbf{j} - 4\mathbf{k}$ perpendicular. [4]

QUESTION 3

(a) The force acting on a particle of mass m is given in terms of time t by

$$\mathbf{F} = a\cos\omega t\mathbf{i} + b\sin\omega t\mathbf{j}.$$

If the particle is initially at rest at the origin, prove that the position at any later time is

$$\underline{\mathbf{r}} = \frac{a}{m\omega^2}(1 - \cos\omega t)\mathbf{i} + \frac{b}{m\omega^2}(\omega t - \sin\omega t)\mathbf{j}.$$

[6]

(b) A lift ascends 380 meters in 2 minutes travelling from rest to rest. For the first 30 seconds it travels with uniform acceleration, for the last 20 seconds with uniform retardation and for the rest of the time it travels with uniform speed. Calculate

(i) the uniform speed in meters per second; [4]

(ii) the uniform acceleration in meters per second squares [5]

(iii) the time taken by the lift to ascend the first 200 meters. [5]

QUESTION 4

(a) A particle of unit mass is thrown vertically upwards with initial speed V, and the air resistance at speed v is κv^2 per unit mass where κ is a constant. Show that H, the maximum height reached, is given by

$$H = \frac{1}{2\kappa} \ln\left(\frac{g + \kappa V^2}{g}\right).$$

[10]

(b) Evaluate

$$\oint (2x - y + 4)dx + (5y + 3x - 6)dy$$

around the triangle in the xy-plane with vertices at (0, 0), (4, 0), (4,2) traversed in the counter-clockwise direction. [10]

QUESTION 5

(a) A particle of mass m moves in a central force field $\mathbf{F} = \frac{K}{r^n}\hat{\mathbf{r}}$ where K and n are constants. It starts from rest at $r = a$ and arrives at $r = 0$ with finite speed v_0. Prove that

$$v_0 = \left\{\frac{2Ka^{1-n}}{m(n-1)}\right\}^{1/2}.$$

[8]

(b) Prove that

$$\nabla \cdot (\phi \mathbf{A}) = (\nabla \phi) \cdot A + \phi(\nabla \cdot \mathbf{A}).$$

[7]

(c) If $\phi = x^2yz^3$ and $\mathbf{A} = xz\mathbf{i} - y^2\mathbf{j} + 2x^2y\mathbf{k}$, find

$$div(\phi \mathbf{A}).$$

[5]

QUESTION 6

A projectile of mass m is launched with initial speed U at an angle θ with the horizontal. If the projectile has acting upon it a force due to air resistance equal to $-\beta\mathbf{v}$, where β is a positive constant and $\mathbf{v}$ is the instantaneous velocity, prove that the position at any time is given by

$$\mathbf{r} = \frac{mU}{\beta}(\cos\theta\mathbf{j} + \sin\theta\mathbf{k})(1 - e^{-\beta t/m}) - \frac{mg}{\beta}(t + \frac{m}{\beta}e^{-\beta t/m} - \frac{m}{\beta})\mathbf{k}.$$

[20]

QUESTION 7

(a) Show, by means of the substitution $r = 1/u$, that the equation of the particle in a central field is

$$\frac{d^2u}{d\theta^2} + u = -\frac{f(1/u)}{mh^2u^2}.$$

[7]

(b) Suppose that a particle mass m is acted upon by a force $\alpha r^{-2} + \beta r^{-3}$ per unit mass where ($\beta = \frac{1}{2}\alpha a$) directed towards the origin $r = 0$ of an inertial frame. Suppose, also, that at $\theta = 0$ and $t = 0$, measurements of distance and velocity of the particle show that it is at distance a from the origin moving with velocity $\sqrt{\alpha/a}$ in a direction perpendicular to the radius vector. If $u = 1/r$, prove that

$$u = \frac{2}{a} - \frac{1}{a}\cos\frac{\theta}{\sqrt{2}}.$$

[13]

END OF EXAMINATION