



# University of Swaziland

Final Examination 2004/2005

B.Sc./B.Ed./B.A.S.S. III

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**Title of Paper** : **Complex Analysis**

**Course Number** : M 313

**Time Allowed** : Three (3) hours

**Instructions** :

1. This paper consists of **seven questions**.
2. Answer **any five questions**.
3. Your work must be accompanied by appropriate explanations.
4. Use of **cellular phones** during the examination is not allowed.
5. Only non-programmable calculators may be used.

**Special requirements:** None

The examination paper must not be opened until permission has been granted by the Invigilator.

Q1.

(a) Solve  $z^2(1 - z^2) = 16$ .

10 [marks]

(b) Represent graphically the set of values of  $z$  for which  $\left|\frac{z-3}{z+3}\right| = 2$ .

10 [marks]

Q2.

If  $z = e^w$  where  $z = r(\cos \theta + i \sin \theta)$  and  $w = u + iv$ , show that  $u = \ln r$  and  $v = \theta + 2k\pi$ ,  $k = 0, \pm 1, \pm 2, \dots$  so that  $w = \ln z = \ln r + i(\theta + 2k\pi)$ .

Determine the values of  $\ln(1 - i)$ .

20 [marks]

Q3.

(a) Using the definition, find the derivative of  $w = f(z) = z^3 - 2z$  at the point where (i)  $z = z_0$  and (ii)  $z = -1$ .

10 [marks]

(b) Show that  $\frac{d\bar{z}}{dz}$  does not exist anywhere.

10 [marks]

Q4.

(a) Prove that  $u = e^{-x}(x \sin y - y \cos y)$  is harmonic.(b) Find  $v$  such that  $f(z) = u + iv$  is analytic.

20 [marks]

Q5.

Evaluate  $\int_{(0,3)}^{(2,4)} (2y + x^2)dx + (3x - y)dy$  along:1. the parabola  $x = 2t, y = t^2 + 3$

2. straight lines from  $(0, 3)$  to  $(2, 3)$  and then from  $(2, 3)$  to  $(2, 4)$
3. a straight line from  $(0, 3)$  to  $(2, 4)$ .

20 [marks]

Q6

- (a) State Cauchy's integral formula.
- (b) Evaluate:

1.  $\int_C \frac{\sin \pi z^2 + \cos \pi z^2}{(z-1)(z-2)} dz$

2.  $\int_C \frac{e^{2z}}{(z+1)^4} dz$  where  $C$  is the circle  $|z| = 3$ .

20 [marks]

Q7.

- (a) Describe how Residue theory can be used for evaluating convergent improper integrals of the form

$$\int_{-\infty}^{\infty} f(x) dx.$$

- (b) Use part (a) to show that:

$$\int_{-\infty}^{\infty} \frac{x^2 dx}{(x^2 + 1)^2(x^2 + 2x + 2)} = \frac{7\pi}{50}$$

20 [marks]

END OF PAPER