178

UNIVERSITY OF SWAZILAND

SUPPLEMENTARY EXAMINATIONS 2005

BSc. / BEd. / B.A.S.S. IV

TITLE OF PAPER	:	FLUID DYNAMICS
COURSE NUMBER	:	M 455
TIME ALLOWED	:	THREE (3) HOURS
INSTRUCTIONS	:	1. THIS PAPER CONSISTS OF SEVEN QUESTIONS. 2. ANSWER ANY FIVE QUESTIONS
SPECIAL REQUIREMENTS	:	NONE

THIS EXAMINATION PAPER SHOULD NOT BE OPENED UNTIL PERMISSION HAS BEEN GRANTED BY THE INVIGILATOR.

QUESTION 1

(a) Dye is continously injected at the point (0, 1, 1) into a fluid with a velocity field

$$\mathbf{q} = (1, y, 2z).$$

Show that the position of the dye streak at later times is given by

$$z = y^2 = e^{2x}$$

[6 marks]

(b) Consider the flow field for an incompressible and irrotational fluid represented by the velocity field

$$\mathbf{q} = -(A + 2Bx)\mathbf{i} + 2By\mathbf{j},$$

Obtain expressions for the velocity potential and streamfunction for the flow.

[8 marks]

(c) Show that the streamlines of the flow corresponding to the complex velocity potential

$$w(z) = \frac{1}{z}$$

satisfy the equation

$$\frac{y}{x^2 + y^2} = \text{constant}$$

[6 marks]

QUESTION 2

(a) The velocity field for an inviscid fluid is given by

$$u = -ay \quad , \quad v = ax \quad , \quad w = 0$$

where a is a constant. Assuming that there are no body forces acting on the fluid,

(i) Prove that the flow is *incompressible*. [3 marks]

(ii) Find the *vorticity* and *rotation* of the fluid. [4 marks]

(iii) If the pressure at $x = y = 0$ is p_0, find an expression for the pressure at each point of the fluid. [5 marks]

(b) A two-dimensional motion of a fluid has a complex potential

$$w(z) = U\left(z + \frac{a^2}{z}\right) + \frac{ik}{2\pi}\log z$$

where U, a and k are constants. Obtain expressions for

(i) the stream function [4 marks]

(ii) the velocity potential [4 marks]

QUESTION 3

(a) A fluid flow had the velocity potential

$$\phi = \frac{\beta x}{x^2 + y^2}$$

(i) Find the velocity components for the flow. [8 marks]

(ii) Show that the flow is both continous and irrotational. [6 marks]

(b) Consider the two-dimensional velocity field

$$\mathbf{q} = \frac{y}{x^2 - 1}\mathbf{i} - \frac{x}{x^2 - 1}\mathbf{j}$$

Calculate the equation of the streamline passing through the point (4,3). [6 marks]

QUESTION 4

(a) State Euler's equation of motion. [4 marks]

(b) Starting from Euler's equation, derive Bernoulli's equation for steady, incompressible flow of potential kind. [8 marks]

(c) Find the complex velocity potential for a two-dimensional irrotational flow with velocity components.

$$u = kx \quad , \quad v = -ky$$

[8 marks]

QUESTION 5

Consider the boundary layer equations in the form

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \bar{U}\frac{d\bar{U}}{dx} + \nu_1\frac{\partial^2 u}{\partial y^2}, \qquad (1)$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \qquad (2)$$

with boundary conditions

$$\begin{aligned} \frac{\partial u}{\partial y} &= 0, & \text{on} \quad y = 0 \\ v &= 0, & \text{on} \quad y = 0 \\ u &= \bar{U}(x). & \text{on} \quad y = \infty \end{aligned}$$

Define

$$\bar{U}(x) = 6\nu_1 x^{-1/3} \quad , \quad \eta = yx^{-2/3} \quad , \quad \psi = -6\nu_1 x^{1/3} F(\eta)$$

where ν_1 is a constant and use the relationships

$$u = -\frac{\partial \psi}{\partial y} \quad , \quad v = \frac{\partial \psi}{\partial x}$$

to show that equation (1) and the boundary conditions (3) transform into

$$F''' + 2FF'' + (F')^2 - 2 = 0$$

$$\begin{aligned} F''(0) &= 0, \\ F(0) &= 0, \\ F'(\infty) &= 1. \end{aligned}$$

where the primes denote differentiation with respect to η. [20 marks]

QUESTION 6

Consider the viscous flow of fluid which is confined between two parallel flat plates of infinite extent in the xy plane. The distance between the plates is 2 with the lower plate fixed at $y = -1$ and the upper plate is located at $y = 1$. The lower plate is held at rest while the upper plate is moving with constant velocity $A\mathbf{i}$. If the velocity field for the flow is

$$\mathbf{q} = (u(y), 0, 0),$$

use the Navier-Stokes equation in the form

$$\frac{\partial \mathbf{q}}{\partial t} + (\mathbf{q} \cdot \nabla)\mathbf{q} = -\frac{1}{\rho}\nabla p + \nu\nabla^2\mathbf{q}$$

(a) to show that the velocity profile for this flow is

$$u(y) = \frac{1}{2\mu}\frac{dp}{dx}\left(y^2 - 1\right) + \frac{A}{2}(y + 1).$$

[15 marks]

(b) Show that the maximum velocity occurs along

$$y = -\frac{A\mu}{2\frac{dp}{dx}}.$$

[5 marks]

QUESTION 7

Water flows out of a reservoir (see Figure 1) down a pipe of cross sectional area, a. Prove that

(a)

$$h = \left\{ (H + h_0)^{1/2} - \frac{1}{2} t \left[(2ga^2)/(A^2 - a^2) \right]^{1/2} \right\}^2 - H$$

[12 marks]

(b) the time to empty the resevoir is **about**

$$\sqrt{\frac{2}{g}} \left\{ (H + h_0)^{1/2} - H^{1/2} \right\} \frac{A}{a}$$

[8 marks]

where h_0 is the depth of the water at $t = 0$, g is the gravity constant and h and H are the heights shown in Figure 1.

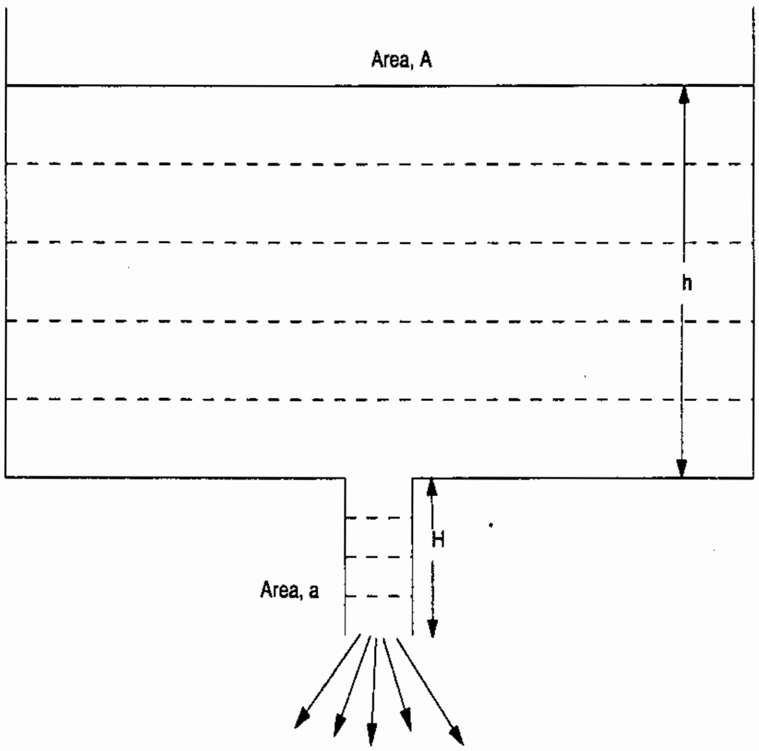

Figure 1: