UNIVERSITY OF SWAZILAND

Final Examination 2006

Title of Paper : Algebra, Trigonometry & Analytic Geometry

Program : BSc./B.Ed. I

Course Number : M 111

Time Allowed : Three (3) Hours

Instructions :

1. This paper consists of seven (7) questions on THREE (3) pages.
2. Answer any five (5) questions.
3. Non-programmable calculators may be used.

Special Requirements : None

THIS EXAMINATION PAPER MAY NOT BE OPENED UNTIL PERMISSION TO DO SO IS GRANTED BY THE INVIGILATOR.

Question 1

(a) (i) Use Cramer's rule to solve the following

$$\begin{aligned} 2x + 8y + z &= 10 \\ 3y - x + 2z &= -2 \\ 4x + 4y - 5z &= 4 \end{aligned}$$

[5 marks]

(ii) Use Gaussian Elimination to solve the following

$$\begin{aligned} x_1 + x_2 + x_3 &= 3 \\ 4x_1 + 5x_2 + 3x_3 &= 11 \\ 5x_1 + 5x_2 + 2x_3 &= 7 \end{aligned}$$

[5 marks]

(b) Find the 6 distinct sixth roots of -64, leaving your answers in the form $a + bi$.
[10 marks]

Question 2

(a) (i) Use synthetic division to find the quotient and remainder if

$(x^3 - 7x^2 - 13x + 3)$ is divided by $(x + 2)$

[5 marks]

(ii) Use long division to find the quotient and remainder if

$(4x^2 - x^2 - 6x - 9)$ is divided by $(2x^2 - x - 3)$

[5 marks]

(b) Prove the following identities

(i) $\sec x - \sin x \tan x = \cos x$

[5 marks]

(ii) $(\tan\theta + \cot\theta)(\cos\theta + \sin\theta) = \csc\theta + \sec\theta$

[5 marks]

Question 3

Give the centre, vertices, foci, eccentricity equations of the directrices and/or the equations of the asymptotes for the following curves

(a) $17 + 30y + 40x - 3y^2 - 4x^2 = 0$

[10 marks]

(b) $4x^2 - 5y^2 - 16x + 10y + 31 = 0$

[10 marks]

Question 4

(a) (i) Find the equation of the parabola with focus $F(-4, 0)$ and directrix $x = 2$.

[5 marks]

(ii) Find the centre and radius of the following circle

$$x^2 + y^2 - 4x - 8y - 16 = 0.$$

[5 marks]

(b) Use mathematical induction to show the following

(i) $1 + 3 + 5 + \cdots + (2n - 1) = n^2$ **[5 marks]**

(ii) $\frac{1}{2} + \frac{1}{2^2} + \frac{1}{2^3} + \cdots + \frac{1}{2^n} = 1 - \frac{1}{2^n}$ **[5 marks]**

Question 5

(a) Solve for z and express in the form $x + yi$

$$z^2 - (3 - i)z + 4 = 0.$$

[10 marks]

(b) If 5, x, y and 32 are in geometric progression, find the values of x and y.

[5 marks]

(c) Find the equation of the line through (1, -2) and parallel to the line through (-1, 4) and (2, 3). **[5 marks]**

Question 6

(a) Prove the following identity

$$\sin^6\theta + \cos^6\theta = 1 - \frac{3}{4}\sin^2 2\theta.$$

[5 marks]

(b) Find the fourth term in the expansion $\left(x^2 - \frac{2}{x}\right)^{-2}$

[5 marks]

(c) Evaluate and express in the form $x + yi$:

(i) $(2\sqrt{3} + i)(4\sqrt{3} - 2i)$ **[5 marks]**

(ii) $\dfrac{2+3i}{-1+i}$ **[5 marks]**

Question 7

(a) Solve for x:

$$\cos 2x + \cos x + 1 = 0.$$

[5 marks]

(b) If the matrix A is given by

$$A = \begin{pmatrix} 1 & 2 & 2 \\ 1 & 3 & 1 \\ 1 & 3 & 2 \end{pmatrix},$$

find the determinant of A and its inverse A^{-1}.

[7 marks]

(c) Given that $\sin\alpha = \dfrac{-3}{5}$ and $\cos\alpha = \dfrac{4}{5}$ find $\sin 2\alpha \tan 2\alpha$. **[4 marks]**

(d) Compute

$$\begin{pmatrix} 1 & 2 & 2 \\ 7 & 3 & 4 \\ 0 & 5 & -6 \end{pmatrix} + \begin{pmatrix} 9 & 8 \\ 5 & 4 \\ -3 & -2 \end{pmatrix}\begin{pmatrix} 3 & -2 & 4 \\ 2 & 1 & 5 \end{pmatrix}.$$

[4 marks]

******** *END OF EXAMINATION* ********