UNIVERSITY OF SWAZILAND

SUPPLEMENTARY EXAMINATIONS 2006

BSc. / BEd. / B.A.S.S. I

TITLE OF PAPER	:	ALGEBRA, TRIG. and ANALYTIC GEOMETRY
COURSE NUMBER	:	M 111
TIME ALLOWED	:	THREE (3) HOURS
INSTRUCTIONS	:	1. THIS PAPER CONSISTS OF SEVEN QUESTIONS.
		2. ANSWER ANY FIVE QUESTIONS
SPECIAL REQUIREMENTS	:	NONE

THIS EXAMINATION PAPER SHOULD NOT BE OPENED UNTIL
PERMISSION HAS BEEN GRANTED BY THE INVIGILATOR.

Question 1

(a) Prove the following identities

    (i) $\dfrac{2\tan\theta}{1+\tan^2\theta} = 2\sin\theta\cos\theta$

    (ii) $\sec^2\theta\csc^2\theta = \sec^2\theta + \csc^2\theta$ **[10 marks]**

(b) (i) If $P(-8,-15)$ is on the terminal side of θ, find $\sin\theta$, $\cos\theta$, $\cos 2\theta$.

    (ii) If $\sin\alpha = -\dfrac{\sqrt{5}}{5}$ and $\cos\alpha = \dfrac{2\sqrt{5}}{5}$, find $\cos 2\alpha$, $\sin 2\alpha$ and $\tan 2\alpha$.

        [10 marks]

Question 2

(a) Evaluate the following

    (i) $(-2+i)-(-8-4i)$

    (ii) $(-4+3i)+(8-2i)$

    (iii) $(-1-i)(-2+3i)$

    (iv) $\dfrac{2+3i}{-1+i}$

        [15 marks]

(b) Write the first six terms of

$$\frac{1}{(1-2x)^2}$$

and simplify using the binomial series.

        [5 marks]

Question 3

(a) (i) Find the equation of a straight line through $(-1,-1)$ that is parallel to the line $2x + 3y + 4 = 0$

(ii) Find the equation of the straight line through $(0,-4)$ that is perpendicular to the line $y + 2 = -\frac{1}{2}(x - 1)$.

[10 marks]

(b) Use mathematical induction to prove that

$$\frac{1}{1 \cdot 2} + \frac{1}{2 \cdot 3} + \cdots + \frac{1}{(n-1)(n)} + \frac{1}{n(n+1)} = \frac{n}{n+1}.$$

[10 marks]

Question 4

(a) Evaluate: $(\sqrt{3} + i)^{12}$.

[5 marks]

(b) Use long division to find the quotient and remainder if

$$(6x^3 - 5x^2 + x - 4) \div (2x^2 - x + 3).$$

[5 marks]

(c) Use synthetic division to find the quotient and remainder if **[5 marks]**

$$(x^4 + 8x + 2) \div (x + 1).$$

(d) Find the three distinct cube roots of 1 leaving your answers in the form $x + yi$.
[5 marks]

Question 5

(a) Draw the circle

$$x^2 + y^2 - 6x + 2y - 26 = 0,$$

indicating its centre and the radius.

[10 marks]

(b) Sketch the parabola

$$y^2 + 6y + 2x + 5 = 0,$$

showing the focus, vertex, axis of symmetry and directrix.

[5 marks]

(c) Find the constant term of

$$\left(2x^2 + \frac{1}{x^2}\right)^{12}.$$

[5 marks]

Question 6

(a) For the ellipse

$$x^2 + 16y^2 + 96y + 128,$$

find the centre, vertices, foci and the directrices. Sketch the curve.

[8 marks]

(b) For the hyperbola

$$4x^2 - 3y^2 - 16x - 18y + 1 = 0$$

find the centre, vertices, foci and the directrices. Sketch the curve.

[7 marks]

(c) Prove the following identity

$$\sin^4 \theta - \cos^4 \theta + \frac{2\cot^2 \theta}{\csc^2 \theta} = 1.$$

[5 marks]

Question 7

(a) (i) Define the ellipse.   [4 marks]

(ii) Find the centre, the foci, and the end point of the major and minor axes for the ellipse

$$\frac{x^2}{25} + \frac{y^2}{16} = 1.$$

[5 marks]

(b) A hyperbola has foci at $(0,0)$ and $(0,4)$. The hyperbola passes through $(12,9)$. Find the general equation of the hyperbola.

[7 marks]

(c) Use Gaussian elimination to solve

$$
\begin{aligned}
x_1 + 2x_2 + 3x_3 &= 6 \\
2x_1 + 5x_2 + x_3 &= 9 \\
x_1 + 4x_2 - 6x_3 &= 1
\end{aligned}
$$

[4 marks]

******** *END OF EXAMINATION* ********