UNIVERSITY OF SWAZILAND

Final Examination 2006

Title of Paper : Introduction to Calculus

Program : BSc./B.Eng./B.Ed./B.A.S.S. I

Course Number : M 115

Time Allowed : Three (3) Hours

Instructions :

1. This paper consists of seven (7) questions on FOUR (4) pages.
2. Answer any five (5) questions.
3. Non-programmable calculators may be used.

Special Requirements : None

THIS EXAMINATION PAPER MAY NOT BE OPENED UNTIL PERMISSION TO DO SO IS GRANTED BY THE INVIGILATOR.

Question 1

(a) Use the definition of the derivative to find $f'(x)$ given that

(i) $f(x) = x^3 + 4x$

(ii) $f(x) = \dfrac{1}{x}$.

[10 marks]

(b) Derive the reduction formula

$$\int \cos^n x \, dx = \frac{\cos^{n-1} x \sin x}{n} + \frac{n-1}{n} \int \cos^{n-2} x \, dx,$$

and use it to evaluate

$$\int \cos^3 x \, dx.$$

[10 marks]

Question 2

(a) Integrate the following

(i) $\displaystyle\int \cos^3 x \sin^4 x \, dx$

(ii) $\displaystyle\int \ln(x^2 + 1) \, dx$

[10 marks]

(b) Find the general expression for $\dfrac{d^n y}{dx^n}$ when

$$y = (x+2)^{\frac{1}{3}}.$$

Here, n is a positive integer.

[10 marks]

Question 3

(a) Evaluate the indefinite integral

$$\int \frac{2x+4}{x^3-2x}\,dx.$$

[10 marks]

(b) Find $\frac{dy}{dx}$ in each of the following

(i) $y = x^{\frac{3}{x}}$

(ii) $y = (\sin x)^{2x}$

[10 marks]

Question 4

(a) Evaluate the following limits

(i) $\lim_{x\to 0}\left(\frac{x^4-3x^3}{x^3}\right)$

(ii) $\lim_{x\to -5}\frac{x+5}{\sqrt{x^2-25}}$

[10 marks]

(b) Integrate the following

(i) $\int \frac{1}{\sqrt{9-25x^2}}\,dx$

(ii) $\int \sin 3x \sin 5x\,dx$

[10 marks]

Question 5

(a) Show that the function

$$y = a\sin cx + b\cos cx,$$

where a, b and c are constants, is a solution of the equation

$$\frac{d^2y}{dx^2} + c^2y = 0.$$

[10 marks]

(b) Integrate the following

(i) $\displaystyle\int \frac{x^{1/2}}{x^{1/3} + x^{1/4}}\, dx$

(ii) $\displaystyle\int \frac{d\theta}{1 + \sin\theta - \cos\theta}\, dx$

[10 marks]

Question 6

(a) Find the area bounded by the curves $y = -x$ and $y = x^2$.

[6 marks]

(b) Use Lebnitz's rule to find $\dfrac{d^4y}{dx^4}$ for

$$y = e^{2x}\sin 4x.$$

[6 marks]

(c) Given that $x = 3(\cos t - t\sin t)$ and $y = 3(\sin t + t\cos t)$, find $\dfrac{dy}{dx}$ in terms of t.

[4 marks]

(d) Find $\dfrac{dy}{dx}$ for $\quad x^4 - y^4 - 4xy = 7.$

[4 marks]

Question 7

(a) Integrate

(i) $\int \arcsin 2x \; dx$,

(ii) $\int x^3 \ln 2x \; dx$.

[10 marks]

(b) Find $\dfrac{dy}{dx}$ when $y = \tanh(e^{-2x})$.

[5 marks]

(c) Find the equation of the line tangent to the curve

$$f(x) = 4 - 3x + 3x^2$$

at the point $(1, 4)$.

[5 marks]

******** *END OF EXAMINATION* ********