UNIVERSITY OF SWAZILAND

FINAL EXAMINATIONS 2006

BSc. / BEd. / B.A.S.S. II

TITLE OF PAPER   :   CALCULUS 1

COURSE NUMBER   :   M 211

TIME ALLOWED   :   THREE (3) HOURS

INSTRUCTIONS   :   1.   THIS PAPER CONSISTS OF
                                            SEVEN QUESTIONS.
                                     2.   ANSWER ANY FIVE QUESTIONS

SPECIAL REQUIREMENTS   :   NONE

THIS EXAMINATION PAPER SHOULD NOT BE OPENED UNTIL
PERMISSION HAS BEEN GRANTED BY THE INVIGILATOR.

QUESTION 1

1. (a) State and prove **Rolle's theorem**   [10 marks]

   (b) Show that for the function $f(x) = \alpha x^2 - \beta x - \phi$ with α, β and ϕ being constants, in any interval $[a, b]$, the value of c guaranteed by the Mean Value Theorem is the mid-point $\frac{a+b}{2}$ of the interval.   [10 marks]

QUESTION 2

2. (a) Sketch the graph of the following function. Indicate **all intercepts, extrema, points of inflection** and **asymptotes** where necessary.

$$f(x) = x^4 - 6x^2$$

[12 marks]

   (b) Use **Newton-Raphson Method** to estimate one of the solutions of the following equation.

$$x^3 + 6 = 2x^2 + 5x$$

Take your $x_0 = 1.5000$. Stop the computation after the **fourth iteration**. Keep every value that you use correct to **4 decimal places** in every part of your working.   [8 marks]

QUESTION 3

3. Evaluate the following limits

   (a) $\displaystyle\lim_{x \to 0} \frac{(x-5)^2 - 25}{x}$ [3 marks]

   (b) $\displaystyle\lim_{x \to 0} \frac{\tan x - x}{x^3}$ [7 marks]

   (c) $\displaystyle\lim_{x \to \infty} \left(\frac{x+1}{x}\right)^x$ [10 marks]

QUESTION 4

4. (a) Use the **Washer Method** to find the volume of the solid obtained by rotating the region bounded by $y = x^2$ and $y = x$ about the line $y = -2$. A sensible sketch of the area and the solid figure is necessary.   [10 marks]

   (b) Using the formula for the arc length with the appropiate sketch, show that the perimeter of a circle of radius a is:

$$C = 2a\pi$$

[10 marks]

QUESTION 5

5. (a) Consider the following sequence

$$a_n = (2^n + 3^n)^{\frac{1}{n}}$$

    i. Is this a decreasing, increasing or non-monotonic sequence? Prove your point.   [5]

    ii. State whether the sequence converges or diverges. If it converges, find the limit. [5]

(b) Show that

$$\lim_{n \to \infty} 2n \sin \frac{\pi}{n} = 2\pi$$

Every step of your method must be clearly shown   [10 marks]

QUESTION 6

6. (a) State (without proof) the following tests for convergence of series:

    i.   The divergence test   [2 marks]

    ii.   The integral test   [3 marks]

    iii.   The alternating series test   [2 marks]

    iv.   The limit comparison test   [3 marks]

(b) Show that the series:

$$\sum_{n=1}^{\infty} \frac{(-1)^n}{2n+1} x^{2n+1}$$

    i.   Converges absulutely for $|x| < 1$ [5 marks]

    ii.   Converges conditionally for $x = 1$ [3 marks]

    iii.   Diverges for $|x| > 1$ [2 marks]

QUESTION 7

7. Use an appropriate test to investigate the following series. Write down the name of the test used.

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{n}$ [4 marks]

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{n}{2^n}$ [4 marks]

(c) $\displaystyle\sum_{n=1}^{\infty} \frac{n!}{n^n}$ [6 marks]

(d) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{\sqrt[3]{8n^2 - 5n}}$ [6 marks]