UNIVERSITY OF SWAZILAND

SUPPLEMENTARY EXAMINATIONS 2006

BSc / B Eng / BEd / B.A.S.S

TITLE OF PAPER	:	CALCULUS I
COURSE NUMBER	:	M211
TIME ALLOWED	:	THREE (3) HOURS
INSTRUCTIONS	:	1. THIS PAPER CONSISTS OF SEVEN QUESTIONS.
		2. ANSWER ANY FIVE QUESTIONS
SPECIAL REQUIREMENTS	:	NONE

THIS EXAMINATION PAPER SHOULD NOT BE OPENED UNTIL PERMISSION HAS BEEN GRANTED BY THE INVIGILATOR.

QUESTION 1

1. State Rolle's theorem. (3)

2. Use your answer from above or otherwise to prove the following theorem:

If f is differentiable on (a,b) and continuous on $[a,b]$, then there is at least one number $c \in (a,b)$ such that $f'(c) = \dfrac{f(b)-f(a)}{b-a}$. *[Mean Value Theorem]* (7)

3. Show that for the function $f(x) = x - x^2$, in any interval $[a,b]$, the value of c guaranteed by the Mean Value Theorem is the mid-point $\dfrac{a+b}{2}$ of the interval.

(10)

20 MARKS

QUESTION 2

1. Sketch the graph of the following function. Indicate *all intercepts, extrema, points of inflection* and *asymptotes* where necessary.

$$f(x) = \frac{2x^2}{x^2-1} \qquad (12)$$

2. Use ***Newton – Raphson Method*** to estimate one of the solutions of the following equation.

$$x^3 = 4x$$

Take your $x_0 = 1.5000$. Stop the computation after the ***fourth iteration***. Keep every value that you use correct to ***4 decimal places*** in every part of your working. (8)

20 MARKS

QUESTON 3

Evaluate the following limits

a. $\lim_{x\to 4}\dfrac{x^3-4x^2+9x-36}{x^2+5}$ (3)

b. $\lim_{x\to 0}\dfrac{\cos x-1+\dfrac{x^2}{2}}{x^4}$ (7)

c. $\lim_{x\to 0}\left(\dfrac{4}{x^2}-\dfrac{2}{1-\cos x}\right)$ (10)

20 MARKS

QUESTION 4

1. Use the ***shell method*** to find the volume of the solid obtained by rotating the region bounded by $y=x-x^2$ and $y=0$ about the line $x=2$. A sensible sketch of the area and the solid figure is necessary. (10)

2. Using the formula for arc length with the appropriate sketch, show that the perimeter of the curved part of a semicircle of radius a is:

$$C=a\pi$$

(10)

20 Marks

QUESTION 5

1. Consider the following sequence

$$a_n = \frac{2n^2 + n}{n^2 + 1}$$

1.1. Is this a decreasing, increasing or non-monotonic sequence? Prove your point. (5)

1.2. State whether the sequence converges or diverges. If it converges, find the limit. (5)

2. Show that

$$\lim_{n\to\infty} \ln\left(n^2 + 1\right)^{\frac{1}{\ln n}} = e^2$$

Every step of your method must be clearly shown. (10)

20 Marks

QUESTION 6

1 By considering even and odd partial sums of the series $\sum_{n=1}^{\infty}(-1)^n a_n$, prove the following theorem.

If $\sum_{n=1}^{\infty}(-1)^{n-1} a_n$ *is an alternating series that satisfies the two conditions:*

$a_{n+1} \le a_n$ *and* $\lim_{n\to\infty} a_n = 0$, *then the series converges.*

(10)

2 Show that the series:

$$\sum_{n=1}^{\infty}\frac{x^n}{n}$$

A Converges absolutely for $|x| < 1$ (3)

B Converges conditionally for $x = -1$ (3)

C Diverges for $x = 1$ and $|x| > 1$ (4)

20 Marks

QUESTION 7

1 Use an appropriate test to investigate the following series. Write down the name of the test used.

A $\sum_{n=1}^{\infty}(-1)^{n-1}\frac{n^3}{3^n}$ (4)

B $\sum_{n=1}^{\infty}\left(\frac{1-3n}{3+4n}\right)^n$ (4)

C $\sum_{n=1}^{\infty}\frac{n^n}{n!}$ (6)

D $\sum_{n=1}^{\infty}\frac{5n-3}{n^2-2n+5}$ (6)

20 Marks