UNIVERSITY OF SWAZILAND

Final Examination 2006

Title of Paper : Calculus II

Program : BSc./B.Eng./B.Ed./B.A.S.S. II

Course Number : M 212

Time Allowed : Three (3) Hours

Instructions :

1. This paper consists of six (6) questions on THREE (3) pages.
2. Answer ANY FIVE questions.
3. Non-programmable calculators may be used.

Special Requirements : None

THIS EXAMINATION PAPER MAY NOT BE OPENED UNTIL PERMISSION TO DO SO IS GRANTED BY THE INVIGILATOR.

Question 1

(a) Find the length of the curve $y = \frac{1}{3}(x^2+2)^{\frac{3}{2}}$ for $0 \leq x \leq 3$.

[9 marks]

(b) Change $(r, \theta) = (2, \frac{3\pi}{2})$ from polar to rectangular coordinates.

[3 marks]

(c) Change $(x, y) = (-1, -1)$ from rectangular to polar coordinates.

[3 marks]

(d) Find an equation of the paraboloid $z = x^2 + y^2$ in spherical coordinates.

[5 marks]

Question 2

(a) Sketch the curves represented by the equations:

(i) $x = t^2 - 2t, \quad y = t + 1$

[7 marks]

(ii) $r = 4 + 4\cos\theta$

[7 marks]

(b) Show that the circumference of a circle with centre (0,0) and radius r is $2\pi r$.

[6 marks]

Question 3

(a) Two objects travel in elliptical paths given by:

$$x_1 = 4\cos\ t \ , \quad y_1 = 2\sin\ t$$

$$x_2 = 2\sin\ 2t \ , \quad y_2 = 3\cos\ t.$$

At what rate is the distance changing between the objects when $t = \pi$?

[10 marks]

(b) Find $\frac{dy}{dx}$ implicitly for the equation $(x+y)^3 + (x-y)^3 = x^4 + y^4$.

[4 marks]

3.(c) Given the ellipse with vertices $(\pm a, 0)$, foci $(\pm c, 0)$ and that $c^2 = a^2 - b^2$ for $a \geq b$ and the useful point $(0, \pm b)$, show that the equation of the ellipse is given by

$$\frac{x^2}{a} + \frac{y^2}{b} = 1$$

if $d_1 + d_2 = 2a$, where d_1 and d_2 are the distances from a point of the ellipse to the foci.

[6 marks]

Question 4

(a) Show that the function $f(x, y) = e^{x \ sin \ y}$ satisfies $\dfrac{\partial^2 f}{\partial x^2} + \dfrac{\partial^2 f}{\partial y^2}$

[8 marks]

(b) If $f(x, y) = e^{xy}$,

(i) Find the rate of change of f at the point P(1,2) in the direction from P to Q(5,4)

(ii) What is the maximum rate of change?

[12 marks]

Question 5

(a) The temperature at a point (x,y) on a metal plate in the x,y-plane is given by $T(x, y) = \dfrac{xy}{1 + x^2 + y^2}$ degrees centigrade.

(i) Find the rate of change of the temperature at (1,1) in the direction of vector $\bar{a} = 2i - j$.

(ii) An ant at (1,1) wants to walk in the direction in which the temperature drops most rapidly. Find a unit vector in that direction.

[12 marks]

(b) Use polar coordinates to evaluate $\int\int (x^2 + y)dA$ over the region between $x^2 + y^2 = 1$ and $x^2 + y^2 = 5$.

[8 marks]

Question 6

(a) Evaluate the iterated integral $\int\int\int r\cos\theta\, dr\, d\theta\, dz$ over the region enclosed by $0 \leq z \leq 4, \quad 0 \leq \theta \leq \frac{\pi}{2}, \quad 0 \leq r \leq 2.$

[8 marks]

(b) Sketch the region whose area is represented by the integral $\int_0^2 \int_{y^2}^4 dxdy$. Then find another iterated integral using the order $dydx$ to represent the same area.

[12 marks]

Question 7

(a) Locate the relative extrema and saddle points for $f(x, y) = 4xy - x^4 - y^4$.

[9 marks]

(b) Find the equation of the tangent plane and normal line to the surface $z = 4x^3y^2 + 2y$ at point P(1, -2, 12).

[6 marks]

(c) Evaluate the area of the region bounded by the graphs $f(x) = \sin x, \quad g(x) = \cos x$ between $x = 0 \quad and \quad x = \dfrac{\pi}{4}$.

[5 marks]

******** *END OF EXAMINATION* ********