# UNIVERSITY OF SWAZILAND

## FINAL EXAMINATIONS 2006

## BSc. / BEd. / B.A.S.S. II

TITLE OF PAPER : ORDINARY DIFFERENTIAL EQUATIONS

COURSE NUMBER : M 213

TIME ALLOWED : THREE (3) HOURS

INSTRUCTIONS : 1. THIS PAPER CONSISTS OF
SEVEN QUESTIONS.
2. ANSWER ANY FIVE QUESTIONS

SPECIAL REQUIREMENTS : NONE

THIS EXAMINATION PAPER SHOULD NOT BE OPENED UNTIL
PERMISSION HAS BEEN GRANTED BY THE INVIGILATOR.

## QUESTION 1

1. (a) Use separation of variables to solve the following differential equations

   i. $(y^2 + y)dx + xdy = 0$ [6 marks]

   ii. $ydx + \dfrac{y-1}{x}dy = 0$ [6 marks]

   (b) Solve the differential equation

$$(x^3 + y^3)dx + xy^2dy = 0$$

[8 marks]

## QUESTION 2

2. (a) Solve the following differential equation [10 marks]

$$(x + y)dx + (3x + 3y - 4)dy = 0$$

   (b) Determine the values of $a$ and $b$ for which the differential equation

$$(3x^2y - y^3 + 2ye^{xy})dx + (x^3 + axy^2 + bxe^{xy})dy = 0$$

   is exact and solve the resulting differential equation. [10 marks]

## QUESTION 3

3. (a) Prove that the differential equation [12 marks]

$$(4xy^2 + 3y)dx + (3x^2y + 2x)dy = 0$$

   has an integrating factor of the form $x^my^n$ and solve the equation.

   (b) Solve the differential equation [8 marks]

$$\frac{dy}{dx} + \frac{3}{x}y = \frac{4}{x^2} + 10x$$

## QUESTION 4

4. Find the general solution of the following homogeneous differential equations

   (a) $y'' - 2y' + 2y = 0$ [6 marks]

   (b) $y'' - 2y' + y = 0$ [6 marks]

   (c) $y'' + \dfrac{y'}{x} - \dfrac{4y}{x^2} = 0$ [8 marks]

## QUESTION 5

5. Find the general solution of the following non-homogeneous differential equations

(a) $\qquad y'' - 3y' + 2y = e^{2x}$ $\hfill$ [5 marks]

(b) $\qquad y'' + y = \sin x$ $\hfill$ [5 marks]

(c) $\qquad y'' + y = \sec x$ $\hfill$ [10 marks]

## QUESTION 6

6. Use Laplace transforms to solve the following differential equations

(a) $\quad y'' + 4y' + 3y = 0 \quad$ subject to $\quad y(0) = 3, \ y'(0) = 1$ $\hfill$ [10 marks]

(b) $\quad y'' + y = 2t \quad$ subject to $\quad y(0) = 0$ and $y'(0) = 0$ $\hfill$ [10 marks]

## QUESTION 7

7. Obtain a power series solution of

$$y' - 2xy = 0$$

about $x = 0$ $\hfill$ [20 marks]