

UNIVERSITY OF SWAZILAND

SUPPLEMENTARY EXAMINATIONS 2006

BSc. / BEd. / B.A.S.S. II

TITLE OF PAPER : ORDINARY DIFFERENTIAL EQUATIONS

COURSE NUMBER : M 213

TIME ALLOWED : THREE (3) HOURS

INSTRUCTIONS : 1. THIS PAPER CONSISTS OF
SEVEN QUESTIONS.
2. ANSWER ANY FIVE QUESTIONS

SPECIAL REQUIREMENTS : NONE

THIS EXAMINATION PAPER SHOULD NOT BE OPENED UNTIL
PERMISSION HAS BEEN GRANTED BY THE INVIGILATOR.

QUESTION 1

1. (a) Use separation of variables to solve the following differential equations

i. $(3x^2 + y^2)dx - 2xydy = 0$ [6 marks]

ii. $y \ln x dx + \frac{y-1}{x} dy = 0$ [6 marks]

- (b) Solve the differential equation

$$y^2 dx + (3xy - x^2) dy = 0$$

[8 marks]

QUESTION 2

2. (a) Solve the following differential equation [10 marks]

$$(x + y)dx + (x + y - 4)dy = 0$$

- (b) Determine the values of a and b for which the differential equation

$$(axy + \cos x + 3e^y)dx + (x^2 + bxe^y)dy = 0$$

is exact and solve the resulting differential equation.

[10 marks]

QUESTION 3

3. (a) Prove that the differential equation [12 marks]

$$(4xy^2 + 3y)dx + (3x^2y + 2x)dy = 0$$

has an integrating factor of the form $x^m y^n$ and solve the equation.

- (b) Solve the differential equation

[8 marks]

$$\frac{dy}{dx} + \frac{1}{x}y = \frac{1}{x}$$

QUESTION 4

4. Find the general solution of the following homogeneous differential equations

(a) $y'' - 2y' + 2y = 0$ [6 marks]

(b) $y'' - 4y' + 4y = 0$ [6 marks]

(c) $2x^2 y'' + 3xy' - y = 0$ [8 marks]

QUESTION 5

5. Find the general solution of the following non-homogeneous differential equations

(a) $y'' - 4y' + 3y = x^2 + x$ [5 marks]

(b) $y'' + 4y = \cos 2x$ [5 marks]

(c) $y'' + y = \sec x$ [10 marks]

QUESTION 6

6. Use Laplace transforms to solve the following differential equations

(a) $y'' + 4y' + 3y = 0$ subject to $y(0) = 3, y'(0) = 1$ [10 marks]

(b) $y'' + y = 2$ subject to $y(0) = 0$ and $y'(0) = 0$ [10 marks]

QUESTION 7

7. Obtain a power series solution of

$$y' - 2xy = 0$$

about $x = 0$

[20 marks]