**UNIVERSITY OF SWAZILAND**

**Final Examination 2006**

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**Title of Paper** : Mathematics for Scientists I

**Program** : BSc./B.Ed. II

**Course Number** : M 215

**Time Allowed** : Three (3) Hours

**Instructions** :

1. This paper consists of seven (7) questions on TWO (2) pages.
2. Answer any five (5) questions.
3. Non-programmable calculators may be used.

**Special Requirements** : None

THIS EXAMINATION PAPER MAY NOT BE OPENED UNTIL PERMISSION TO DO SO IS GRANTED BY THE INVIGILATOR.

**Question 1**

(a) Evaluate the integral

$$\iint_R (x+2y)\,dx\,dy,$$

where R is the triangle bounded by the lines $y = x$, $y = 1 - x$, and the y-axis.

**[8 marks]**

(b) Evaluate the following limits:

(i) $\displaystyle\lim_{x\to 0} \frac{1+\cos x}{\sin 2x}$		(ii) $\displaystyle\lim_{x\to 0} \left(\frac{1}{x} - \frac{1}{\sin x}\right)$

**[12 marks]**

**Question 2**

(a) Find the first four nonzero terms of the Maclaurin's series of the function $f(x) = \dfrac{1}{1-x}$. Hence, deduce the first four nonzero terms of the Maclaurin's series of $g(x) = \dfrac{1}{1+x^2}$.

**[10 marks]**

(b) Locate all relative extrema and saddle points of $f(x) = x^2 + 2y^2 - x^2 y$.

**[10 marks]**

**Question 3**

(a) Reverse the order of integration and evaluate the resulting integral:

$$\int_0^9 \int_{\sqrt{y}}^3 \sin x^3 \; dx \; dy.$$

**[10 marks]**

(b) Find the general solution of the differential equation

$$y^2 \; dx + (x^2 - 2xy) \; dy = 0.$$

**[10 marks]**

**Question 4**

(a) Use differentials to find an approximate value of: $7.8\,(26)^{1/3}$.

**[10 marks]**

(b) Solve the differential equation: $y''+y'+2y=0$.

**[10 marks]**

**Question 5**

(a) Use the method of Lagrange multipliers to find extreme values of $f(x,y)=x^2-y$, subject to the constraint $x^2+y^2=25$.

**[10 marks]**

(b) Solve the following differential equation: $(3x^2+y\cos x)\,dx+(\sin x-4y^3)\,dy=0$.

**[10 marks]**

**Question 6**

(a) Let R be the region in the first quadrant between the circles $x^2+y^2=1$ and $x^2+y^2=9$. Evaluate

$$\iint_R e^{-(x^2+y^2)}dx\,dy.$$

**[10 marks]**

(b) For the function $f(x)=\sqrt{x+1}$ in [0, 3], verify that the hypotheses of the Mean Value Theorem are satisfied, and find the number $c$ in (0, 3) whose existence is guaranteed by the theorem.

**[10 marks]**

**Question 7**

(a) Let $V=V\left(\frac{x}{z}\right)$. Deduce that $xV_x+yV_y+zV_z=0$.

**[10 marks]**

(b) Given the vectors $\mathbf{a}=\hat{\mathbf{i}}-\hat{\mathbf{j}}+2\hat{\mathbf{k}}$ and $\mathbf{b}=2\hat{\mathbf{i}}+\hat{\mathbf{j}}+\hat{\mathbf{k}}$, find a unit vector that is orthogonal to both $\mathbf{a}$ and $\mathbf{b}$.

**[10 marks]**

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