UNIVERSITY OF SWAZILAND

FINAL EXAMINATIONS 2006

BSc. / BEd. / B.A.S.S. II

TITLE OF PAPER	:	DYNAMICS I
COURSE NUMBER	:	M 255
TIME ALLOWED	:	THREE (3) HOURS
INSTRUCTIONS	:	1. THIS PAPER CONSISTS OF SEVEN QUESTIONS. 2. ANSWER ANY FIVE QUESTIONS
SPECIAL REQUIREMENTS	:	NONE

THIS EXAMINATION PAPER SHOULD NOT BE OPENED UNTIL PERMISSION HAS BEEN GRANTED BY THE INVIGILATOR.

QUESTION 1

(a) In cylindrical coordinates (r, θ, z), the position vector of an arbitrary point (x, y, z) is given by

$$\mathbf{R} = r\cos\theta\hat{\mathbf{i}} + r\sin\theta\hat{\mathbf{j}} + z\hat{\mathbf{k}}$$

Show that, in this coordinate system

(i) the velocity is given by

$$\underline{\mathbf{v}} = \frac{d\mathbf{R}}{dt} = \dot{r}\hat{\mathbf{r}} + r\dot{\theta}\hat{\theta} + \dot{z}\hat{\mathbf{k}}$$

[6 marks]

(ii) the acceleration is given by

$$\underline{\mathbf{a}} = \frac{d\underline{\mathbf{v}}}{dt} = (\ddot{r} - r\dot{\theta}^2)\hat{\mathbf{r}} + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\hat{\theta} + \ddot{z}\hat{\mathbf{k}}$$

[4 marks]

(b) Find the equation of the plane that contains the point $(2, 1, 0)$ and has a normal vector $\mathbf{n} = (1, 2, 3)$. [6 marks]

(c) For what values of a are $A = a\mathbf{i} - 2\mathbf{j} + \mathbf{k}$ and $B = 2a\mathbf{i} + a\mathbf{j} - 4\mathbf{k}$ perpendicular. [4 marks]

QUESTION 2

(a) The force acting on a particle of mass m is given in terms of time t by

$$\mathbf{F} = a\cos\omega t\mathbf{i} + b\sin\omega t\mathbf{j}$$

If the particle is initially at rest at the origin, prove that the position at any later time is

$$\underline{\mathbf{r}} = \frac{a}{m\omega^2}(1-\cos\omega t)\mathbf{i} + \frac{b}{m\omega^2}(\omega t - \sin\omega t)\mathbf{j}$$

[6 marks]

(b) A lift ascends 400 metres in 2 minutes traveling from rest to rest. For the first 30 seconds it travels with uniform acceleration, for the last 20 seconds with uniform retardation and for the rest of the time it travels with uniform speed. Calculate

(i) the uniform speed in metres per second; [4 marks]

(ii) the uniform acceleration in metres per second squares [5 marks]

(iii) the time taken by the lift to ascend the first 200 metres. [5 marks]

QUESTION 3

(a) A particle of unit mass is thrown vertically upwards with initial speed V, and the air resistance at speed v is κv^2 per unit mass where κ is a constant. Show that H, the maximum height reached, is given by

$$H = \frac{1}{2\kappa}\ln\left(\frac{g+\kappa V^2}{g}\right)$$

[10 marks]

(b) Evaluate

$$\oint (2x-y^2)dx + (3y-4x)dy$$

around the triangle in the xy-plane with vertices at (0, 0), (2,1), (2,0) traversed in the clockwise direction. [10 marks]

QUESTION 4

(a) Suppose that a point A has position vector $\mathbf{a}$ and a point B has position vector $\mathbf{b}$. Show that the position vector $\mathbf{r}$ of the point R that divides the line AB in the ratio $\alpha : \beta$ is given by

$$\mathbf{r} = \frac{\beta\mathbf{a} + \alpha\mathbf{b}}{\alpha + \beta}.$$

Hence, deduce the midpoint formula. [7 marks]

(b) If $\mathbf{r}(t) = \mathbf{a}\cos(\omega t) + \mathbf{b}\sin(\omega t)$, where $\mathbf{a}$ and $\mathbf{b}$ are constant non-collinear vectors and ω is a constant scalar, prove that

(i) $\mathbf{r} \times \frac{\mathrm{d}\mathbf{r}}{\mathrm{d}t} = \omega(\mathbf{a} \times \mathbf{b})$

(ii) $\frac{\mathrm{d}^2\mathbf{r}}{\mathrm{d}t^2} + \omega^2\mathbf{r} = \mathbf{0}$. [8 marks]

(c) If $\phi = x^2yz^3$ and $\mathbf{A} = xz\mathbf{i} - y^2\mathbf{j} + 2x^2y\mathbf{k}$, find

$$div(\phi\mathbf{A})$$

[5 marks]

QUESTION 5

If

$$\mathbf{r}(t) = 4\sin t\hat{\mathbf{i}} + 4\cos t\hat{\mathbf{j}} + 8\hat{\mathbf{k}},$$

find

(a) the unit tangent vector $\hat{\mathbf{T}}$ [5 Marks]

(b) the curvature κ [5 Marks]

(c) the unit principal normal $\hat{\mathbf{N}}$ [5 Marks]

(d) the unit binormal vector $\hat{\mathbf{B}}$. [5 Marks]

QUESTION 6

(a) (i) Show that $\mathbf{A} = (6xy^2 - y^3)\hat{\mathbf{i}} + (6x^2y - 3xy^2)\hat{\mathbf{j}}$ is a conservative vector field, and hence find the value of the integral $I = \int_{(1,2)}^{(3,4)} \mathbf{A}.\mathbf{dr}$. [7 marks]

(ii) If $\mathbf{F} = (2x + y)\hat{\mathbf{i}} + (3x - 2y)\hat{\mathbf{j}}$, find $\int_C \mathbf{F}.\mathbf{dr}$,where C is the straight line from $(0, 0)$ to $(2, 2)$. [4 marks]

(b) Verify Green's theorem in the plane for the vector field $(x^2 - xy^3)\hat{\mathbf{i}} + (y^2 - 2xy)\hat{\mathbf{j}}$ for a square with vertices $(0, 0)$, $(2, 0)$, $(2, 2)$, $(0, 2)$. [9 marks]

QUESTION 7

(a) From a point O, at height h above sea level, a particle is projected under gravity with a velocity of magnitude $\frac{3}{2}\sqrt{gh}$. Find the two possible angles of projection if the particle strikes the sea at horizontal distance $3h$ from 0. [10 marks]

(b) Two points A and B are at distance d apart. A particle starts from A and moves in the direction $\overrightarrow{AB}$ with initial velocity u and uniform acceleration a. A second particle starts at the same time from B and moves in the direction $\overrightarrow{BA}$ with initial velocity $2u$ and retardation a.

(i) Prove that the particles collide at time $\frac{d}{3u}$ from the beginning of the motion. [5 marks]

(ii) Prove that if the particles collide before the second particle returns to B, then

$$ad < 12u^2.$$

[5 marks]