UNIVERSITY OF SWAZILAND

FINAL EXAMINATION 2006/7

B.Sc. I,B.Ed I, B.Eng I

TITLE OF PAPER: Algebra, Trigonometry and Analytic Geometry

COURSE NUMBER: M111

TIME ALLOWED: THREE HOURS

INSTRUCTIONS:

1. This paper consists of SEVEN questions on FOUR pages.
2. Answer any FIVE questions.
3. Calculators may be used.

SPECIAL REQUIREMENTS: NONE

THIS EXAMINATION PAPER SHOULD NOT BE OPENED UNTIL PERMISSION HAS BEEN GRANTED BY THE INVIGILATOR.

Question 1

(a) Evaluate the following and express the results in the form $a + bi$

(i) $(8 - 2i)(-4 + 3i)$ **[5 marks]**

(ii) $\dfrac{2 + 3i}{-1 + i}$ **[5 marks]**

(b) Find the equations of the following lines and express your answer in the form $y = mx + c$

(i) Line through $(-1, 4)$ which is parallel to the line through $(1, -1)$ and $(2, 3)$ **[5 marks]**

(ii) Line through $(-1, -2)$ which is perpendicular to the line

$$2x + 3y - 4 = 0$$

[5 marks]

Question 2

(a) (i) Given that $\sin A = \frac{3}{5}$ and $\cos A$ is negative.
Find $\sin 2A$ and $\tan 2A$ **[5 marks]**

(ii) Prove the following identity

$$\frac{2\tan\theta}{1 + \tan^2\theta} = 2\sin\theta\cos\theta$$

[5 marks]

(b) (i) Find the equation of the parabola with focus $F(-4, 0)$ and directrix $x = 2$ **[5 marks]**

(ii) Find the centre and radius of the following circle

$$x^2 + y^2 - 4x - 8y - 16 = 0$$

[5 marks]

Question 3

(a) (i) Find the first five terms of the infinite binomial expansion

$$(1+2x)^{-2}$$

[5 marks]

(ii) Find the middle term of the binomial expansion

$$\left(2x^2+\frac{1}{x}\right)^{12}$$

[5 marks]

(b) Find the centre, vertices, foci, eccentricity, equations of the directrices for the following hyperbola and sketch the curve.

$$4x^2-5y^2-16x+10y+31=0$$

[10 marks]

Question 4

(a) Given the following matrices

$$A=\begin{pmatrix}-3 & 5\\ 2 & -1\end{pmatrix} \qquad A=\begin{pmatrix}-5 & 1 & 2\end{pmatrix}$$

$$C=\begin{pmatrix}1 & 3\\ -1 & 0\\ 4 & -2\end{pmatrix} \qquad D=\begin{pmatrix}-2 & 9 & 6\\ -3 & 3 & 4\\ 2 & -2 & 1\end{pmatrix} \qquad E=\begin{pmatrix}3 & -2 & 4\\ 2 & 1 & 5\end{pmatrix}$$

Compute

(i) AE

(ii) BC

(iii) E^T+C

(iv) BE^T

(v) $|D|$

[10 marks]

(b) Use mathematical induction to prove that

(i) $1+3+5+\cdots+(2n-1)=n^2$ **[5 marks]**

(ii) $2.2^1+3.2^2+4.2^3+\cdots+(n+1)2^n=n2^{n+1}$ **[5 marks]**

Question 5

(a) (i) Use Cramer's rule to solve the following

$$\begin{aligned} 2x + 8y + z &= 10 \\ -x + 3y + 2z &= -2 \\ 4x + 4y - 5z &= 4 \end{aligned}$$

(ii) Use Gaussian Elimination to solve the following

$$\begin{aligned} x_1 + x_2 + x_3 &= 3 \\ 4x_1 + 5x_2 + 3x_3 &= 11 \\ 5x_1 + 5x_2 + 2x_3 &= 7 \end{aligned}$$

[10 marks]

(b) Find the distinct sixth roots of -64, leaving all your answers in the form $a + bi$

[10 marks]

Question 6

(a) (i) Use long division to find the quotient and remainder

$$(4x^4 - x^2 - 6x - 9) \div (2x^2 - x - 3)$$

[5 marks]

(ii) Use synthetic division to find the quotient and remainder

$$(x^3 - 7x^2 - 13x + 3) \div (x + 2)$$

[5 marks]

(b) Prove the following identities

(i) $(\tan\theta + \cot\theta)(\cos\theta + \sin\theta) = \csc\theta + \sec\theta$ **[5 marks]**

(ii) $\sec x - \sin x \tan x = \cos x$

[5 marks]

Question 7

(a) (i) Find the centre, foci, directrices and the end points of the major and minor axes for the ellipses

$$\frac{x^2}{25} + \frac{y^2}{169} = 1$$

[7 marks]

(b) A hyperbola has foci at $(0,0)$ and $(0,4)$ and passes through the point $(12,9)$. Find the equation of the hyperbola.

[8 marks]

(c) Compute $(1+i)^{20}$ using deMoivre's theorem.

[5 marks]

******** *END OF EXAMINATION* ********