University of Swaziland

Final Examination, May 2007

BSc I, Bass I, BEd I, BEng I

Title of Paper : Introduction to Calculus
Course Number : M115
Time Allowed : Three (3) hours
Instructions :

1. This paper consists of SEVEN questions.
2. Each question is worth 20%.
3. Answer ANY FIVE questions.
4. Show all your working.

THIS PAPER SHOULD NOT BE OPENED UNTIL PERMISSION HAS BEEN GIVEN BY THE INVIGILATOR.

Question 1

(a) Find $f'(x)$ using the limit definition of the derivative, given that

$$f(x) = \frac{1}{1+x^2}. \qquad [12]$$

(b) Find y' and simplify if

$$\ln(x^2+y^2) + \arctan\frac{x}{y} = 1. \qquad [8]$$

Question 2

(a) Show that the function $\rho(x,y) = \sqrt{x^2+y^2}$ satisfies

$$\frac{\partial^2 \rho}{\partial x^2} + \frac{\partial^2 \rho}{\partial y^2} = \frac{1}{\rho}. \qquad [10]$$

(b) Find the exact value of the area bounded by the curves $y = 5x - x^2$ and $y = 3x^2 - x$. [10]

Question 3

(a) Find y'' and simplify, given

$$y = \sqrt{2 - 2x + x^2}. \qquad [10]$$

(b) Integrate

$$\int 2x^4 \cosh 2x \, dx. \qquad [10]$$

Question 4

(a) Use an appropriate method to evaluate

$$\int e^{1-2x} \cos x \, \mathrm{d}x. \qquad [10]$$

(b) Find

$$\int_2^4 \sqrt{4x - x^2} \, \mathrm{d}x. \qquad [10]$$

Question 5

(a) Use Leibnitz's rule to evaluate

$$\frac{\mathrm{d}^4}{\mathrm{d}x^4}\left(\frac{1}{xe^{2x}}\right). \qquad [10]$$

(b) Integrate

$$\int_0^1 x^2\sqrt{1-x} \, \mathrm{d}x. \qquad [10]$$

Question 6

(a) Work out

$$\int \frac{(2x+1)\mathrm{d}x}{x^3 + x^2 + x + 1}. \qquad [12]$$

(b) Use mathematical induction to prove the formula

$$\frac{\mathrm{d}^n}{\mathrm{d}x^n}(x^{n-1} \ln x) = \frac{(n-1)!}{x}, \qquad [8]$$

where n is an integer greater than zero.

Question 7

A rectangular cardboard of sides 10 m by 20 m is to be used to construct an open box by cutting out small squares of equal size at the corners, and folding up the edges. If the squares cut out are of side x m,

(a) show that the volume of the box is given by $V(x) = 4x^3 - 60x^2 + 200x$. [5]

(b) Find the value of x that yields the maximum volume, and hence show that the maximum volume is $\frac{1000}{9}\sqrt{3}\,\mathrm{m}^3$. [15]

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