

University of Swaziland

Supplementary Examination, July 2007

BSc I, Bass I, BEd I

Title of Paper : Introduction to Calculus

Course Number : M115

Time Allowed : Three (3) hours

Instructions :

1. This paper consists of SEVEN questions.
2. Each question is worth 20%.
3. Answer ANY FIVE questions.
4. Show all your working.

THIS PAPER SHOULD NOT BE OPENED UNTIL PERMISSION HAS BEEN GIVEN BY THE INVIGILATOR.

Question 1

- (a) Find $f'(x)$ using the limit definition of the derivative, given that

$$f(x) = 2 + \sqrt{x}. \quad [12]$$

- (b) Find y' and simplify if

$$y = (x - 1)\sqrt{x^2 - 2x + 2}. \quad [8]$$

Question 2

- (a) Use Leibnitz's rule to find

$$\frac{d^4}{dx^4} \left\{ \frac{\ln x}{2x} \right\}. \quad [10]$$

- (b) Work out

$$2x^3 \sin 2x \, dx. \quad [10]$$

Question 3

- (a) Integrate

$$\int \frac{x \, dx}{x^2 - x - 6}. \quad [12]$$

- (b) For the function $f(x, y) = \ln \sqrt{x^2 + y^2}$, find $\frac{\partial^2 f}{\partial x^2}$. [8]

Question 4

- (a) Integrate

$$\int \frac{x^4 - x^3 - x + 1}{x^3 - x^2} dx. \quad [10]$$

- (b) Given that the curve $y = x^3 + ax^2 + bx + c$ passes through the point $(1, 1)$ and has stationary points at $x = -1$ and $x = 3$, find the values of a , b and c . [10]

Question 5

(a) Integrate

$$\int_0^2 \sqrt{4-x^2} dx. \quad [12]$$

(b) Given the function $R(t) = A \sin 2t + B \cos 2t$, where A and B are constants, show that

$$\frac{d^2R}{dt^2} + 4R = 0. \quad [8]$$

Question 6

(a) Work out

$$\int_0^{\frac{\pi}{2}} \sin^4 \theta d\theta. \quad [12]$$

(b) A rectangular field is to be bounded by a fence on 3 sides and a straight existing wall on the fourth. Find the dimensions of the field of maximum area that can be enclosed using 1000 m of fence. [8]

Question 7

(a) Evaluate

$$\int_0^{\ln 2} \frac{d\theta}{1+e^\theta}. \quad [12]$$

(b) Find y' given

$$x^2 = \frac{x+y}{x-y}. \quad [8]$$

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