UNIVERSITY OF SWAZILAND

FINAL EXAMINATION 2007

BSC. II,B.Ed II, BASS II

TITLE OF PAPER: CALCULUS II

COURSE NUMBER: M212

TIME ALLOWED: THREE HOURS

INSTRUCTIONS:

1. This paper consists of SEVEN questions on FOUR pages.
2. Answer any FIVE questions.
3. Non-programmable calculators may be used

SPECIAL REQUIREMENTS: NONE

THIS EXAMINATION PAPER SHOULD NOT BE OPENED UNTIL PERMISSION HAS BEEN GRANTED BY THE INVIGILATOR.

## Question 1

(a) Sketch the curve represented by

$$x = \frac{t}{t+1}, \qquad y = \frac{t^2}{1+t} \quad \text{for} \quad \frac{-2}{3} \leq t \leq \frac{-4}{3}$$

**[10 marks]**

(b) Change $(r, \theta), = (2, \frac{7\pi}{4})$ from polar to rectangular.

**[3 marks]**

(c) Change $(x, y) = (-1, -1)$ to polar coordinates

**[3 marks]**

(d) Find the rectangular coordinates of $(\rho, \theta, \varphi) = (4, \frac{\pi}{3}, \frac{\pi}{4})$

[4 marks]

## Question 2

(a) Find the arc length of $y = x^{\frac{2}{3}}$ between $x = -1$ and $x = 8$

**[7 marks]**

(b) Sketch the graph of $r = 2 - 2\cos\theta$

**[8 marks]**

(c) Show that for $u(x, y) = e^x \sin y$

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0.$$

**[5 marks]**

## Question 3

(a) Locate all relative extrema and saddle points of

$$f(x,y) = 4xy - x^4 - y^4$$

[7 marks]

(b) Use the chain rule to find $W_t$ and $W_u$ if

$$W = x\ln(x^2+y^2), \quad x = t+u, \quad y = t-u$$

**[10 marks]**

(c) Use differentials to approximate

$$26^{\frac{1}{3}}$$

[3 marks]

## Question 4

(a) Find the gradient of the surface

$$f(x,y) = \frac{-x^2}{4} - y^2 + \frac{25}{8}xy \quad \text{at} \quad (\frac{1}{2}, 2)$$

**[5 marks]**

(b) Find the equation of the tangent plane and normal line to the surface

$$Z = 4x^3y^2 + 2y \quad \text{at} \quad (1,-2,6)$$

**[6 marks]**

(c) Use the method of Lagrange multipliers to find the extrema values of

$$f(x,y) = x^2 - xy + y^2 - 3x \quad \text{subject to} \quad x^2 + y^2 = a$$

**[9 marks]**

## Question 5

(a) Sketch and evaluate the area of the region represented by

$$\int\int_R dxdy$$

where $R$ is defined by

$$R = \{y^2 \leq x \leq 4, \quad 0 \leq y \leq 2\}$$

**[8 marks]**

(b) Find another iterated integral using the order $dydx$ (reversed order) and show that both integrals yield the same area.

**[6 marks]**

(c) If $Z$ is defined implicity by

$$(x+y)^3 - (x-y)^3 = x^4 + y^4,$$

find $\frac{dy}{dx}$

**[6 marks]**

## Question 6

(a) Use polar coordinates to evaluate

$$\int\int_R (x^2+y)dA$$

over the region

$$x^2 + y^2 = 4, \qquad x^2 + y^2 = 9.$$

**[12 marks]**

(b) Find the area of the region $R$ that lies below the parabola $y = 4x - x^2$ and above the line $y = 4 - x$

**[8 marks]**

## Question 7

(a) Find the directional derivative of $f(x, y) = 3x^2 - 2y^2$ at $(\frac{-3}{4}, 0)$ in the direction from $P$ to $a(0, 1)$

[6 marks]

(b) Suppose the temperature of a point $(x, y, z)$ is given by

$$T(x, y, z) = \frac{80}{1 + x^2 + 2y^2 + 3z^2}$$

degrees celcius
(i) in what direction does the temperature increase fastest at the point $(1, 1, -2)$
(ii) What is this maximum increase?

**[6 marks]**

(c) Evaluate the area enclosed by the graphs $f(x) = \sin x, f(x) = \cos x$ for $0 \leq x \leq \frac{\pi}{4}$

[8 marks]

******** *END OF EXAMINATION* ********