University of Swaziland

Final Examination, May 2007

BSc II, Bass II, BEd II, BEng II

Title of Paper : Ordinary Differential Equations

Course Number : M213

Time Allowed : Three (3) hours

Instructions :

1. This paper consists of SEVEN questions.

2. Each question is worth 20%.

3. Answer ANY FIVE questions.

4. Show all your working.

THIS PAPER SHOULD NOT BE OPENED UNTIL PERMISSION HAS
BEEN GIVEN BY THE INVIGILATOR.

Question 1

(a) Find the general solution of

$$(y^2 - 1)dx - 2(2y + xy)dy = 0. \qquad \text{[8 marks]}$$

(b) Use Laplace transforms to solve

$$\ddot{y} + y = t, \ y(0) = 1, \ \dot{y}(0) = 0$$

where $\dot{y} = \frac{dy}{dt}$. \qquad [12 marks]

Question 2

Solve each of the following equations

(a) $\quad xdy - (y + x^3 e^x)dx = 0, \ y(1) = 0$ \qquad [8 marks]

(b) $\quad y'' + 4y' + 4y = e^{-2x}.$ \qquad [12 marks]

Question 3

Use the method of Frobenius to find a series solution of

$$4xy'' + 2y' + y = 0$$

about $x = 0$. \qquad [20 marks]

Question 4

(a) Your friend claims

> All *first order* ODEs can be made *exact* by using a standard *integrating factor*.

Is your friend right or wrong? Discuss. In your discussion, define all terms shown in italics.

[7 marks]

(b) Find the general solution of

    (i) $y^{iv} + 5y'' - 36y = 0$ [6 marks]

    (ii) $x dy + (y - x^3 y^6) dx = 0$ [7 marks]

Question 5

(a) Find the general solution of

$$y'' + 2y' + 5y = 0.$$ [8 marks]

(b) Use Laplace transforms to solve

$$\ddot{y} + 2\dot{y} + 5y = 0, \quad y(0) = \dot{y}(0) = 1.$$ [12 marks]

Question 6

(a) Find the general solution of

$$(y - x + 5)dy - (y - x + 1)dx = 0.$$ [8 marks]

(b) Find the solution of

$$y'' + 2y' - 15y = 30e^{-2x}$$

that satisfies the conditions $y(0) = 2, \ y'(0) = 0$.

                                               [12 marks]

Question 7

(a) Use *two different methods* to solve

$$2xy dy + (x^2 - y^2)dx = 0.$$ [15 marks]

(b) Obtain the general solution of

$$2x^2 y'' - 3xy' - 3y = 0.$$ [5 marks]

.... END OF PAPER ....