UNIVERSITY OF SWAZILAND

FINAL EXAMINATIONS 2006/7

BSc. II

TITLE OF PAPER	:	MATHEMATICS FOR SCIENTISTS
COURSE NUMBER	:	M 215
TIME ALLOWED	:	THREE (3) HOURS
INSTRUCTIONS	:	1. THIS PAPER CONSISTS OF SEVEN QUESTIONS. 2. ANSWER ANY FIVE QUESTIONS
SPECIAL REQUIREMENTS	:	NONE

THIS EXAMINATION PAPER SHOULD NOT BE OPENED UNTIL PERMISSION HAS BEEN GRANTED BY THE INVIGILATOR.

Question 1

(a) If $\underline{a} = \hat{\mathbf{i}} - \hat{\mathbf{j}} + 2\hat{\mathbf{k}}$, $\underline{b} = 2\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}$, $\underline{c} = (1, 2, -1)$. Verify that

$$\underline{b}(\underline{b} \cdot \underline{a}) + \underline{a}(\underline{c} \cdot \underline{b}) = -[(\underline{a} \times \underline{b}) \times \underline{c}]$$

[6]

(b) Find an equation in rectangular coordinates of the surface whose equation in cylindrical coordinates is $r = 4\cos\theta$

[4]

(c) Find the first 6 non-zero terms of the Taylor series of $f(x) = \sin x$ about $x = 0$. Use this series to approximate

$$\int_0^2 \sin x^2 dx$$

to 4 decimal places. Also use the series to deduce the first 4 nonzero terms of $h(x) = \sin 2x$. [10]

Question 2

(a) Prove that if $f(x)$ is continuous on $[a, b]$ and differentiable on (a, b) then there is a point $c \in (a, b)$ such that $f'(c) = 0$ if $f(b) = f(a)$. [10]

(b) Verify that for $f(x) = \sqrt{x-1}$ in $[1, 5]$ the Mean Value Theorem hypothesis is satisfied. [4]

(c) Evaluate the limits

(i) $\lim_{x \to 0} x^3 \ln x$ [3]

(ii) $\lim_{x \to \pi} \frac{\ln \cos 2x}{(\pi - x)^2}$ [3]

Question 3

(a) Locate all relative extrema and saddle points for $f(x,y) = 4xy - x^4 - y^4$ [9]

(b) Let $z = xye^{\frac{x}{y}}$ with $x = r\cos\theta$, $y = r\sin\theta$. Find $\dfrac{dz}{d\theta}$ when $\theta = \frac{\pi}{6}$ an d $r = 2$ [6]

(c) Solve the differential equation

$$xydx + (x^2+1)dy = 0$$

[5]

Question 4

(a) Use the method of Langrange Multipliers to find the extrema values of $f(x,y) = x^2 - xy + y^2 - 3x$ subject to $x^2 + y^2 = 9$ [12]

(b) If $f(x,y) = xe^{x^2y}$ find $\dfrac{\partial f}{\partial x}$ and $\dfrac{\partial f}{\partial y}$ at $(2, ln2)$ [8]

Question 5

(a) Find the general solution of the differential equation.

$$(x^3+y^3)dx + xy^2dy = 0$$

[10]

(b) Evaluate the area of the region bounded by
$f(x) = \sin x$ and $f(x) = \cos x$ between $x = 0$ and $x = \frac{\pi}{4}$ [6]

(c) If $f(x,y) = x^2y^2 + x^4y$, show that $\dfrac{\partial^2 f}{\partial x \partial y} = \dfrac{\partial^2 f}{\partial y \partial x}$. [4]

Question 6

(a) Use polar coordinates to evaluate the double integral $\int\int_R (x^2 + y)dA$ where R is the region enclosed by $x^2 + y^2 = 1$ and $x^2 + y^2 = 5$ [10]

(b) Solve the differential equation

$$y''' - y'' - 10y' - 8y = 0$$

[10]

Question 7

(a) Sketch and evaluate the area of the region represented by $\displaystyle\int\int_R dxdy$ in

$$R = \{y^2 \le x \le 4, \quad 0 \le y \le z\}.$$

Find another iterated integral in the order $dydx$ to show the same area. [10]

(b) Evaluate the triple integral

$$\int\int\int_R r\cos\theta dr d\theta dz$$

over

$0 \le z \le 4, \quad 0 \le r \le 2, \quad 0 \le \theta \le \frac{\pi}{2}$ [6]

(c) Use the differentials to find the approximate value of $3\sqrt{64.08}$ to 4 decimal places. [4]

******** *END OF EXAMINATION* ********