UNIVERSITY OF SWAZILAND

SUPPLEMENTARY EXAMINATIONS 2007

BSc. II

TITLE OF PAPER	:	MATHEMATICS FOR SCIENTISTS
COURSE NUMBER	:	M 215
TIME ALLOWED	:	THREE (3) HOURS
INSTRUCTIONS	:	1. THIS PAPER CONSISTS OF SEVEN QUESTIONS. 2. ANSWER ANY FIVE QUESTIONS
SPECIAL REQUIREMENTS	:	NONE

THIS EXAMINATION PAPER SHOULD NOT BE OPENED UNTIL PERMISSION HAS BEEN GRANTED BY THE INVIGILATOR.

Question 1

(a) Find a unit vector perpendicular to both $\underline{a} = (2, -6, -3)$ and $\underline{b} = (4, 3, -1)$ [4]

(b) Find the first 4 non zero terms of the Maclaurin's series of

$$f(x) = \sin x.$$

Hence deduce the first 4 terms of the Maclaurin's series of

$$h(x) = \sin 2x.$$

[10]

(c) If $\underline{a} = (1, 1, 1)$ $\underline{b} = (2, 3, 2)$ and θ is the angle between the vectors find

(i) $\sin\theta$

(ii) $\cos\theta$

(iii) Confirm that $\sin^2\theta + \cos^2\theta = 1$. [2,2,2]

Question 2

(a) Prove Rolle's Theorem [10]

(b) Let $f(x) = x^4 - 2x^2$. Find $c \in (-2, 2)$ such that $f'(c) = 0$ [5]

(c) Find $\lim_{x \to 0} \left(\frac{1}{x} - \frac{1}{\ln(x+1)} \right)$ [5]

Question 3

(a) Use implicit differentiation to find $\dfrac{dy}{dx}$ if

$$x^4 + y^4 = (x+y)^3 - (x-y)^3$$

[5]

(b) Use the chain rule to find w_t and w_u if

$$w = x\ln(x^2+y^2)\,, \quad x = t+u, \quad y = t-u$$

[10]

(c) Find the equation in cylindrical coordinates of the surface whose equation in rectangular coordinates is

$$z = x^2 + y^2 - 2x + y$$

[5]

Question 4

(a) Locate all extreme points and saddle points of

$$f(x,y) = x^3 + y^3 - 3x - 12xy + 20$$

[10]

(b) Use Lagrange Multipliers to find the extreme values of

$$f(x,y,z) = 9xy + 16yz + 16xz \quad \text{subject to} \quad xyz = 16$$

[10]

Question 5

(a) Evaluate the iterated integral

$$\int_0^1 \int_2^1 \left(\frac{x}{(xy+1)^2}\right) dydx$$

[5]

(b) Use double integral to evaluate the area enclosed by the parabola $y = \frac{1}{2}x^2$ and the line $y = 2x$. Reverse the order of integration and show that the area is the same. [10]

(c) Solve the differential equation

$$y'' - 2y' + 2y = 0$$

[5]

Question 6

(a) Solve the differential equation

$$y''' - 4y'' - y' + 4y = 0$$

[10]

(b) Use polar coordinates to evaluate the integral

$$\int\int_R \sqrt{x^2 + y^2} dxdy$$

over the region enclosed by $x^2 + y^2 = 4$ and $x^2 + y^2 = 9$. [10]

Question 7

(a) Solve the differential equation

$$(x^3 + y^3)dx + xy^2dy = 0$$

[10]

(b) Evaluate

$$\int\int\int_R 24xy^2z^3dxdydz$$

where R is the region in the rectangular box

$$0 \leq x \leq a, \quad 0 \leq y \leq b, \quad 0 \leq z \leq c.$$

[5]

(c) Use differentials to find the approximate value of $(26)^{\frac{1}{3}}$ [5]

******** *END OF EXAMINATION* ********