UNIVERSITY OF SWAZILAND

FINAL EXAMINATIONS 2006/2007

B.Sc. / B.Ed. / B.A.S.S. II

TITLE OF PAPER	:	DYNAMICS I
COURSE NUMBER	:	M255
TIME ALLOWED	:	THREE (3) HOURS
INSTRUCTIONS	:	1. THIS PAPER CONSISTS OF SEVEN QUESTIONS. 2. ANSWER ANY FIVE QUESTIONS
SPECIAL REQUIREMENTS	:	NONE

THIS EXAMINATION PAPER SHOULD NOT BE OPENED UNTIL PERMISSION HAS BEEN GRANTED BY THE INVIGILATOR.

QUESTION 1

(a) Prove that

$$\nabla \cdot (\phi \mathbf{A}) = (\nabla \phi) \cdot A + \phi(\nabla \cdot \mathbf{A})$$

[6]

(b) Let $\mathbf{A} = (2xy + z^3)\,\hat{\mathbf{i}} + x^2\,\hat{\mathbf{j}} + 3xz^2\,\hat{\mathbf{k}}$ be a given vector field.

(i) Show that $\mathbf{A}$ is conservative.

(ii) Find the scalar potential.

(iii) Find the work done in moving an object in this field from from $(1, -2, 1)$ to $(3, 1, 4)$. [5,7,2]

QUESTION 2

The position vector of a moving particle is given by

$$\mathbf{r} = 3\cos(2t)\hat{\mathbf{i}} + 3\sin(2t)\hat{\mathbf{j}} + (8t - 4)\hat{\mathbf{k}}.$$

Find

(a) the velocity

(b) the speed

(c) the acceleration

(d) the magnitude of the acceleration

(e) the unit tangent vector

(f) the curvature

(g) the radius of curvature

(h) the unit principal normal

(i) the normal component of acceleration

(j) the unit binormal vector. [20]

QUESTION 3

(a) In cylindrical coordinates (s, θ, z), the position vector of an arbitrary point (x, y, z) is given by

$$\mathbf{r} = s\cos\theta\hat{\mathbf{i}} + s\sin\theta\hat{\mathbf{j}} + z\hat{\mathbf{k}}.$$

Show that, in this coordinate system,

(i) the velocity is given by

$$\underline{\mathbf{v}} = \frac{d\mathbf{r}}{dt} = \dot{s}\hat{\mathbf{s}} + s\dot{\theta}\hat{\theta} + \dot{z}\hat{\mathbf{k}}$$

(ii) the acceleration is given by

$$\mathbf{a} = \frac{d\mathbf{v}}{dt} = (\ddot{s} - s\dot{\theta}^2)\hat{\mathbf{s}} + (s\ddot{\theta} + 2\dot{s}\dot{\theta})\hat{\theta} + \ddot{z}\hat{\mathbf{k}}.$$

[10,2]

(b) Find the work done in moving a particle once around a circle in the xy-plane if the circle has its center at the origin, has radius 3, and if the vector field is given by $\mathbf{F} = (2x - y + z)\,\hat{\mathbf{i}} + (x + y - z^2)\,\hat{\mathbf{j}} + (3x - 2y + 4z)\,\hat{\mathbf{k}}$. [8]

QUESTION 4

(a) If $\mathbf{A} = (3x^2 - 6yz)\,\hat{\mathbf{i}} + (2y + 3xz)\,\hat{\mathbf{j}} + (1 - 4xyz^2)\,\hat{\mathbf{k}}$, evaluate $\int_C \mathbf{A}.\mathbf{dr}$ from $(0,0,0)$ to $(1,1,1)$ along the following paths C:

(i) $x = t$, $y = t^2$, $z = t^3$.

(ii) The straight line from $(0,0,0)$ to $(0,0,1)$, then to $(0,1,1)$, and then to $(1,1,1)$.

(iii) The straight line joining $(0,0,0)$ and $(1,1,1)$. [4,5,3]

(b) Show that $\oint_C \dfrac{x\mathrm{d}y - y\mathrm{d}x}{x^2 + y^2} = 2\pi$, where C is the circle $x^2 + y^2 = a^2$ traversed in the counterclockwise direction. [8]

QUESTION 5

(a) A particle moves in a central force field defined by

$$\mathbf{F} = -\frac{K}{r^2}\hat{\mathbf{r}},$$

where K is a constant and r is the distance from the center of force (origin).

(i) Find the potential energy of the particle.

(ii) Find the amount of work done by the force field in moving the particle from a point on the circle of radius $r = a(> 0)$ to another point on the circle of radius $r = b(> 0)$. [3,3]

(b) Evaluate $\oint_C (y - \sin x)\mathrm{d}x + \cos x\mathrm{d}y$, where C is the triangle in the xy-plane with vertices at $(0,0)$, $(\pi/2, 0)$, $(\pi/2, 1)$ traversed in the counterclockwise direction:

(i) Directly.

(ii) Using Green's Theorem. [8,6]

QUESTION 6

(a) A particle is projected with velocity $\mathbf{u}$ from a point O in a vertical plane through the line of greatest slope of a plane inclined at an angle β to the horizontal. After time T, the particle strikes the inclined plane at the point P, at a distance R from O. If $\mathbf{u}$ makes an angle α with the horizontal, and if $|\mathbf{u}| = u$, show that:

(i) $T = \dfrac{2u\sin(\alpha-\beta)}{g\cos\beta}$ and $R = \dfrac{u^2[sin(2\alpha-\beta)-\sin\beta]}{g\cos^2\beta}$;

(ii) for constant u and β, R is maximum when $\alpha = \dfrac{\pi}{4} + \dfrac{\beta}{2}$. [9,3]

(b) Under the influence of a central force, a particle moves in a circular orbit through the origin. Find the law of force. [8]

QUESTION 7

(a) A particle is projected from the origin with initial velocity $-4\hat{\mathbf{i}}$ and acceleration $(3-t)\hat{\mathbf{i}}$, where t is measured in seconds. Show that the particle reverses direction after 2 seconds and after 4 seconds. [5]

Also find the total distance traveled by the particle in

(i) 2 seconds

(ii) 4 seconds

(iii) 6 seconds [1,1,1]

(b) At time $t = 0$ a particle of mass m is located at $z = 0$ and is traveling vertically downwards with speed v_0.If the resisting force is $-\beta v$, where v is the speed at time t, find

(i) the speed at any time t,

(ii) the distance traveled after time t, and

(iii) the acceleration at any time t. [6,4,2]

END OF EXAMINATION