UNIVERSITY OF SWAZILAND

SUPPLEMENTARY EXAMINATIONS 2006/2007

B.Sc. / B.Ed. / B.A.S.S. II

TITLE OF PAPER	:	DYNAMICS I
COURSE NUMBER	:	M255
TIME ALLOWED	:	THREE (3) HOURS
INSTRUCTIONS	:	1. THIS PAPER CONSISTS OF SEVEN QUESTIONS. 2. ANSWER ANY FIVE QUESTIONS
SPECIAL REQUIREMENTS	:	NONE

THIS EXAMINATION PAPER SHOULD NOT BE OPENED UNTIL PERMISSION HAS BEEN GRANTED BY THE INVIGILATOR.

QUESTION 1

(a) Evaluate $\int_{(0,1)}^{(1,2)} [(x^2 - y)\,\mathrm{d}x + (y^2 + x)\,\mathrm{d}y]$ along:

(i) the straight line from $(0,1)$ to $(1,2)$;

(ii) the straight lines from $(o,1)$ to $(1,1)$ and then from $(1,1)$ to $(1,2)$;

(iii) the parabola $x = t, \quad y = t^{+}1$. [10]

(b) Show that $\oint_C \frac{x\mathrm{d}y - y\mathrm{d}x}{x^2 + y^2} = 2\pi$, where C is the circle $x^2 + y^2 = a^2$ traversed in the counterclockwise direction. [10]

QUESTION 2

The position vector of a moving particle is given by

$$\mathbf{r} = 2\cos(t)\hat{\mathbf{i}} + 2\sin(t)\hat{\mathbf{j}} + (2t - 1)\hat{\mathbf{k}}.$$

Find

(a) the velocity

(b) the speed

(c) the acceleration

(d) the magnitude of the acceleration

(e) the unit tangent vector

(f) the curvature

(g) the radius of curvature

(h) the unit principal normal

(i) the normal component of acceleration

(j) the unit binormal vector. [20]

QUESTION 3

(a) Verify Green's theorem in the plane for the vector field $(x^2 - xy^3)\hat{\mathbf{i}} + (y^2 - 2xy)\hat{\mathbf{j}}$ for a square with vertices $(0,0)$, $(2,0)$, $(2,2)$, $(0,2)$. [10 marks]

(b) Find the equation of the plane that contains the point $(2,1,0)$ and has a normal vector $\mathbf{n} = (1,2,3)$. [6 marks]

(c) For what values of a are $A = a\mathbf{i} - 2\mathbf{j} + \mathbf{k}$ and $B = 2a\mathbf{i} + a\mathbf{j} - 4\mathbf{k}$ perpendicular. [4 marks]

QUESTION 4

(a) In cylindrical coordinates (ρ, θ, z), the position vector of an arbitrary point (x, y, z) is given by

$$\mathbf{r} = \rho\cos\theta\hat{\mathbf{i}} + \rho\sin\theta\hat{\mathbf{j}} + z\hat{\mathbf{k}}.$$

Show that, in this coordinate system,

(i) the velocity is given by

$$\underline{\mathbf{v}} = \frac{d\mathbf{r}}{dt} = \dot{\rho}\hat{\rho} + \rho\dot{\theta}\hat{\theta} + \dot{z}\hat{\mathbf{k}}$$

(ii) the acceleration is given by

$$\mathbf{a} = \frac{\mathrm{d}\mathbf{v}}{\mathrm{d}t} = (\ddot{\rho} - \rho\dot{\theta}^2)\hat{\rho} + (\rho\ddot{\theta} + 2\dot{\rho}\dot{\theta})\hat{\theta} + \ddot{z}\hat{\mathbf{k}}.$$

[10,2]

(b) If $\nabla\phi = 2xyz^3\hat{\mathbf{i}} + x^2z^3\hat{\mathbf{j}} + 3x^2yz^3\hat{\mathbf{k}}$ and if $\phi(1,-2,2) = 4$, find $\phi(x,y,z)$. [8]

QUESTION 5

(a) A particle of unit mass is thrown vertically upwards with initial speed V, and the air resistance at speed v is κv^2 per unit mass where κ is a constant. Show that H, the maximum height reached, is given by

$$H = \frac{1}{2\kappa}\ln\left(\frac{g+\kappa V^2}{g}\right)$$

[10 marks]

(b) From a point O, at height h above sea level, a particle is projected under gravity with a velocity of magnitude $\frac{3}{2}\sqrt{gh}$. Find the two possible angles of projection if the particle strikes the sea at horizontal distance $3h$ from 0. [10 marks]

QUESTION 6

(a) A particle is projected from the origin with initial velocity $-4\hat{\mathbf{i}}$ and acceleration $(3-t)\hat{\mathbf{i}}$, where t is measured in seconds. Show that the particle reverses direction after 2 seconds and after 4 seconds. [5]

Also find the total distance traveled by the particle in

(i) 2 seconds

(ii) 4 seconds

(iii) 6 seconds [1,1,1]

(b) At time $t = 0$ a particle of mass m is located at $z = 0$ and is traveling vertically downwards with speed v_0.If the resisting force is $-\beta v$, where v is the speed at time t, find

(i) the speed at any time t,

(ii) the distance traveled after time t, and

(iii) the acceleration at any time t. [6,4,2]

QUESTION 7

((a) Suppose that a point A has position vector $\mathbf{a}$ and a point B has position vector $\mathbf{b}$. Show that the position vector $\mathbf{r}$ of the point R that divides the line AB in the ratio $\alpha : \beta$ is given by

$$\mathbf{r} = \frac{\beta\mathbf{a} + \alpha\mathbf{b}}{\alpha + \beta}.$$

Hence, deduce the midpoint formula. [7 marks]

(b) If $\mathbf{r}(t) = \mathbf{a}\cos(\omega t) + \mathbf{b}\sin(\omega t)$, where $\mathbf{a}$ and $\mathbf{b}$ are constant non-collinear vectors and ω is a constant scalar, prove that

(i) $\mathbf{r} \times \frac{\mathrm{d}\mathbf{r}}{\mathrm{d}t} = \omega(\mathbf{a} \times \mathbf{b})$

(ii) $\frac{\mathrm{d}^2\mathbf{r}}{\mathrm{d}t^2} + \omega^2\mathbf{r} = \mathbf{0}$. [8 marks]

(c) If $\phi = x^2yz^3$ and $\mathbf{A} = xz\mathbf{i} - y^2\mathbf{j} + 2x^2y\mathbf{k}$, find

$$div(\phi\mathbf{A})$$

[5 marks]

END OF EXAMINATION