University of Swaziland

Supplementary Examination, 2007

BSc IV, Bass IV, BEd IV

Title of Paper : Partial Differential Equations

Course Number : M415

Time Allowed : Three (3) hours

Instructions :

1. This paper consists of SEVEN questions.
2. Each question is worth 20%.
3. Answer ANY FIVE questions.
4. Show all your working.
5. Useful formulae are attached at the end of the paper.

THIS PAPER SHOULD NOT BE OPENED UNTIL PERMISSION HAS BEEN GIVEN BY THE INVIGILATOR.

Question 1

(a) State the order of each of the following PDEs, and classify as linear or non-linear. [5 marks]

(i) $u_{xxxx} + yu = 4\tan(y)$

(ii) $xu_{xx} + 2xyu_{xy} - y^2u = 0$

(iii) $u_x^2 + u_y^2 = u$

(iv) $uu_{xx} - yu_{yyy} = \tan u$

(v) $\frac{\partial^2 u}{\partial x^2} = \frac{1}{k}\frac{\partial u}{\partial t}$

(b) Find a particular solution of $u_{xx} = 1$ that satisfies the conditions $u(1, y) = y^2$ and $u_x(1, y) = y$. [15 marks]

Question 2

(a) Classify the equation $4u_{xx} - 4u_{xy} + u_{yy} = 2$ and reduce it into its canonical form. Hence find the general solution. [15 marks]

(b) Differentiate $u(x, y) = f(x + 2y) + g(x - 2y)$, where f and g are arbitrary functions, to show that it satisfies the PDE $4u_{xx} - u_{yy} = 0$. [5 marks]

Question 3

(a) Find a particular solution of the PDE $4u_x + 8u_y - u = 1$ that satisfies $u = 2e^{x/4} - 1$ on $y = 2x + 2$. [14 marks]

(b) Eliminate the arbitrary function in $u = xy + f(x^2 + y^2)$ to obtain the PDE satisfied by $u(x, y)$. [6 marks]

Question 4

Solve the non-homogeneous Cauchy problem for the wave equation [20 marks]

$$\begin{aligned} u_{tt} &= c^2u_{xx} + x + ct, \quad -\infty < x < \infty,\ t \geq 0, \\ u(x, 0) &= x, \quad -\infty < x < \infty, \\ u_t(x, 0) &= \sin x, \quad -\infty < x < \infty. \end{aligned}$$

Question 5

Find the solution of the steady-state problem [20 marks]

$$\begin{aligned} u_{xx}+u_{yy} &= 0, \quad 0\le x,y\le\pi, \\ u(0,y) &= 8\sin^3 y, \; 0\le y <\le \pi, \\ u(\pi,y) &= 0, \qquad 0\le y<\le\pi, \\ u(x,0) &= u(x,\pi), \; 0\le x\le\pi. \end{aligned}$$

Question 6

Solve [20 marks]

$$\begin{aligned} u_t &= u_{xx}, \quad 0<x<\pi, \; t>0, \\ u(x,0) &= \sin x + \tfrac{1}{3}\sin 3x, \quad 0\le x\le\pi, \\ u(0,t) &= u(\pi,t)=0, \quad t>0. \end{aligned}$$

Question 7

Use Laplace transforms to solve [20 marks]

$$\begin{aligned} u_{xx}-\frac{1}{c^2}u_{tt}+\sin\pi x &= 0, \quad 0\le x\le 1, \; t>1, \\ u(x,0)=u_t(x,0) &= 0, \; 0\le x\le 1, \\ u(0,t)=u(1,t) &= 0, \qquad t\ge 0. \end{aligned}$$

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