UNIVERSITY OF SWAZILAND

FINAL EXAMINATIONS 2007

BSc. / BEd. / B.A.S.S. IV

TITLE OF PAPER	:	FLUID DYNAMICS
COURSE NUMBER	:	M 455
TIME ALLOWED	:	THREE (3) HOURS
INSTRUCTIONS	:	1. THIS PAPER CONSISTS OF SEVEN QUESTIONS. 2. ANSWER ANY FIVE QUESTIONS
SPECIAL REQUIREMENTS	:	NONE

THIS EXAMINATION PAPER SHOULD NOT BE OPENED UNTIL PERMISSION HAS BEEN GRANTED BY THE INVIGILATOR.

QUESTION 1

1. (a) Dye is continuously injected at the point (0,1,1) into a fluid with a velocity field $\mathbf{q} = (1, y, 2z)$. Show that the position of the dye streak at later times is given by $z = y^2 = e^{2x}$. [6 marks]

   (b) A three-dimensional velocity distribution is given by $u = -x$, $v = 2y$ and $w = 6 - z$. Show that the equation of the stream line through (1, 2, 3) is

$$x^2 y = 2 \quad , \quad \frac{(6-z)}{x} = 3$$

[6 marks]

   (c) Verify that the velocity field

$$\mathbf{q} = (5t - 2x)\mathbf{i} + 2y\mathbf{j}$$

could describe the flow of an incompressible fluid. If the fluid is inviscid and the body force per unit mass is $\mathbf{F} = 5\mathbf{i}$, show that

$$p = A(t) - 2\rho(x^2 - 5tx + y^2)$$

where $A(t)$ is an arbitrary function of t. [8 marks]

QUESTION 2

2. (a) Verify that the velocity field $\mathbf{q} = x^2\mathbf{i} - 2xy\mathbf{j}$ could describe the flow of an incompressible fluid. If the fluid is inviscid and the body force per unit mass is $\mathbf{F} = -xy^2\mathbf{i} + x^2y\mathbf{j}$, show that the pressure distribution is given by [8 marks]

$$p = p_0 - \frac{\rho}{2}x^2(x^2 + y^2)$$

(b) Find the equation of the streamlines corresponding to the following velocity field

$$\mathbf{q} = \left\{ U\left(1 - \frac{a^2}{r^2}\right)\cos\theta, -U\left(1 + \frac{a^2}{r^2}\right)\sin\theta, 0 \right\}$$

[8 marks]

(c) Find the stream function corresponding to the velocity field given by

$$\mathbf{q} = x^2y^2\mathbf{i} - xy^3\mathbf{j}$$

if it exists. [4 marks]

QUESTION 3

3. (a) Find the velocity potential ϕ corresponding to the stream function

$$\psi = Ur\sin\theta\left(1 - \frac{a^2}{r^2}\right)$$

[6 marks]

(b) For the complex velocity potential given by

$$w(z) = U\left(z + \frac{a^2}{z}\right) + \frac{ik}{2\pi}\ln z$$

find the

i. stream function [2 marks]

ii. velocity potential [2 marks]

iii. velocity components [4 marks]

(c) Find the complex velocity potential $w(z)$ corresponding to the velocity potential

$$\phi = (x+1)^2 - y^2$$

[6 marks]

QUESTION 4

4. (a) Fluid flows out of a circular tank of radius A though a small circular hole of radius a located in the bottom of the tank. Assuming that the flow is steady and that the pressure both at the free surface and the exit hole is atmospheric, show that the time T required to empty the tank is given by

$$T = \left[\frac{A^4}{a^4} - 1\right]^{\frac{1}{2}} \left(\frac{2h_0}{g}\right)^{\frac{1}{2}}$$

where the height $h = h_0$ when $t = 0$ and g is acceleration due to gravity.

(b) Two doublets, each of strength μ, are situated at the points $z = \pm a$, and two other doublets, each of strength 2μ are situated at the points $z = \pm ia$. Each of the four doublets has its axis tangential to the circle $|z| = a$ and pointing in the positive sense of rotation. Show that $w(z)$, the total complex velocity potential of the four doublets is given by

$$w(z) = \frac{2\mu ia(3a^2 - z^2)}{z^4 - a^4}$$

QUESTION 5

5. (a) Find the constants a and b if

$$\phi = ax^3y + bxy^3$$

is a potential function. Hence show that the stream function corresponding to ϕ is [10 marks]

$$\psi = \frac{3}{2}ax^2y^2 - \frac{1}{4}a(x^4 + y^4) + c$$

(b) Consider the viscous flow of fluid which is confined between two parallel flat plates of infinite extent in the xy plane. The distance between the plates is 2 with the lower plate fixed at $y = -1$ and the upper plate is located at $y = 1$. The lower plate is held at rest while the upper plate is moving with constant velocity $A\mathbf{i}$. If the velocity field for the flow is

$$\mathbf{q} = (u(y), 0, 0),$$

use the Navier-Stokes equation in the form

$$\frac{\partial \mathbf{q}}{\partial t} + (\mathbf{q} \cdot \nabla)\mathbf{q} = -\frac{1}{\rho}\nabla p + \nu \nabla^2 \mathbf{q}$$

to find the velocity profile. [10 marks]

QUESTION 6

6. (a) An infinite row of line vortices, each of strength $m > 0$, are placed at the points

$$z = 0, \pm a, \pm 2a, \pm 3a, \ldots, \pm na$$

where a is a real positive number. Use the following infinite product identity

$$\sin\frac{\pi z}{a} = \frac{\pi z}{a}\left(1 - \frac{z}{a}\right)\left(1 + \frac{z}{a}\right)\left(1 - \frac{z}{2a}\right)\left(1 + \frac{z}{2a}\right)\cdots$$

to show that the complex potential for the flow is

$$w(z) = \frac{im}{2\pi}\log\sin\frac{\pi z}{a} \quad + \quad \text{constant}$$

[12 marks]

(b) Find the velocity components and show that

$$(u, v) \to \left(-\frac{m}{2a}, 0\right) \quad \text{as} \quad y \to +\infty$$
$$(u, v) \to \left(+\frac{m}{2a}, 0\right) \quad \text{as} \quad y \to -\infty$$

[8 marks]

QUESTION 7

7. Consider the boundary layer equations in the form [20 marks]

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (1)$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu\frac{\partial^2 u}{\partial y^2} - au + au\frac{T - T_\infty}{T_w - T_\infty} \quad (2)$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \nu\frac{\partial^2 T}{\partial y^2} \quad (3)$$

with boundary conditions

$$u = ax \;,\; v = -(\nu a)^{\frac{1}{2}} \;\text{ on y} = 0 \;\text{ and }\; u = 0 \;\text{ on }\; y = \infty$$

$$T = T_w \quad \text{on} \quad y = 0 \quad \text{and} \quad T = T_\infty \quad \text{on} \quad y = \infty$$

Using the similarity transformation $\eta = \left(\frac{a}{\nu}\right)^{\frac{1}{2}} y$, $\theta = \frac{T - T_\infty}{T_w - T_\infty}$ and the stream function formulation $\psi = -x(\nu a)^{\frac{1}{2}} f(\eta)$ where a, T_w and T_∞ are constants and ν is the dynamic viscosity, Show that equations (2) and (3) and the boundary conditions can be transformed into

$$f''' + ff'' - (f')^2 - f' = 0$$

$$\theta'' + f\theta' = 0$$

$$f = 1 \quad , \quad f' = 1 \;\text{ on }\; \eta = 0 \quad \text{and} \quad f' = 0 \quad \text{on } \eta = \infty$$

$$\theta = 1 \;\text{ on }\; \eta = 0 \quad \text{and} \quad \theta = 0 \quad \text{on } \eta = \infty$$