University of Swaziland

Final Examination – December 2007

BSc I, EEng I, BEd I

Title of Paper : Algebra, Trig. and Analytic Geometry
Course Number : M111
Time Allowed : Three (3) hours
Instructions :

1. This paper consists of SEVEN questions.
2. Each question is worth 20%.
3. Answer ANY FIVE questions.
4. Show all your working.

THIS PAPER SHOULD NOT BE OPENED UNTIL PERMISSION HAS BEEN GIVEN BY THE INVIGILATOR.

Question 1

(a) Work out

$$\frac{x^4 - 2x^3 - 2x - 20}{2 - x^2}.$$ [10 marks]

(b) Find the constant term of the binomial expansion of $\left(x - \frac{2}{\sqrt{x}}\right)^{14}$. [6 marks]

(c) Find the value of the infinite sum $2 - \sqrt{2} + 1 - \cdots$. [4 marks]

Question 2

(a) Consider the parametric equation

$$x = 1 - 2\sin\theta, \; y = 2 - 5\cos\theta.$$

i. Eliminate θ and write the equation in terms of x and y. [4 marks]

ii. Describe the curve defined by the equation and hence make a sketch. [6 marks]

(b) Find the fourth roots of

$$-8 - 8i\sqrt{3}.$$ [10 marks]

Question 3

(a) Find the values of A and B such that $x^2 - 9$ is a factor of $P(x) = x^3 + Ax^2 + Bx + 4$. [6 marks]

(b) Find the first five terms of the binomial expansion of

$$\frac{1 + x^2}{\sqrt{1 - 2x^2}}.$$ [7 marks]

(c) Prove by mathematical induction

$$1 + 3 + 3^2 + \cdots + 3^{n-1} = \tfrac{1}{2}(3^n - 1). \qquad \text{[7 marks]}$$

Question 4

(a) Given the complex numbers
$\alpha = 1 - i\sqrt{3},\ \lambda = -\sqrt{3} + i,\ \kappa = -2i$, evaluate

i) $\alpha\lambda$ ii) $\dfrac{\alpha}{\lambda}$ iii) $\dfrac{\alpha\kappa}{\lambda^2}$ iv) $\left(\dfrac{\alpha}{2}\right)^{20}$ [12 marks]

(b) Find all values of x in the interval $0 \leq x < 2\pi$, satisfying

$$\sin x + \cos 2x = 0. \qquad \text{[8 marks]}$$

Question 5

(a) A section of a stadium has 50 rows of seats. The first rows has 28 seats, the second row has 32 seats, and so on, increasing by 4 seats each row. Rows number 25 to 50 are sheltered from rain and sunshine. Find the total number of seats sheltered. [4 marks]

(b) Solve

i. $2^{3-2x} = 5 \cdot 3^{x-1}$ [4 marks]

ii. $\log_2(x-4) + \log_2(x+3) = 3$ [4 marks]

(c) Prove

$$\frac{\sin A + \sin 2A}{1 + \cos A + \cos 2A} = \tan A. \qquad \text{[8 marks]}$$

Question 6

(a) Find all roots of

$$x^4 + x^3 - 13x^2 + 12 = 0. \qquad \text{[10 marks]}$$

(b) Find the equation of the circle(s) tangent to the line $2x - 3y + 1 = 0$ at $(1, 1)$ with radius $r = \sqrt{13}$.

[10 marks]

Question 7

(a) Use Cramer's rule to solve the system

$$\begin{aligned} x + y + z &= 3 \\ 2x + y - z &= -6 \\ 3x - y + z &= 11. \end{aligned}$$

[14 marks]

(b) Use mathematical induction to prove

$$\cos\left(\varphi - n\pi\right) = \left(-1\right)^n \cos\varphi,$$

where n is an integer. [6 marks]