

---

# University of Swaziland



## Supplementary Examination 2008

---

### BSc I, EEng I, BEd I

**Title of Paper** : Algebra, Trig. and Analytic Geometry

**Course Number** : M111

**Time Allowed** : Three (3) hours

**Instructions** :

1. This paper consists of SEVEN questions.
2. Each question is worth 20%.
3. Answer ANY FIVE questions.
4. Show all your working.

THIS PAPER SHOULD NOT BE OPENED UNTIL PERMISSION HAS BEEN GIVEN BY THE INVIGILATOR.

### Question 1

(a) Work out

$$\frac{4x^4 + 12x^3 - 21x - 20}{x + \frac{1}{2}}. \quad [10 \text{ marks}]$$

(b) Expand  $\left(x^2 - \frac{2}{\sqrt{x}}\right)^5$  and simplify term by term. [10 marks]

### Question 2

(a) Find three numbers in arithmetic progression such that their sum is 15 and their product is 80. [10 marks]

(b) Find the cube roots of

$$-27i. \quad [10 \text{ marks}]$$

### Question 3

(a) Find the values of  $A$  and  $B$  such that  $x + 3$  is a factor of  $P(x) = Ax^3 + 3x^2 + Bx - 12$ , and  $P(x)$  leaves a remainder of  $-6$  when divided by  $x + 1$ . [6 marks]

(b) Find the first five terms of the binomial expansion of

$$(1 + x)\sqrt{1 - 2x}. \quad [7 \text{ marks}]$$

(c) Prove that the sum of the first  $n$  positive integers is  $n^2$ . [7 marks]

#### Question 4

(a) Given the complex numbers

$\alpha = 3 - 4i$ ,  $\lambda = -4 + 3i$ , evaluate

i)  $3\alpha - 4\lambda$     ii)  $\alpha\lambda$     iii)  $\frac{\alpha}{\lambda}$     iv)  $|\alpha|$     [10 marks]

(b) Find all values of  $x$  in the interval  $0 \leq x < 2\pi$ , satisfying

$$\cos^2 x - \sin x + 1 = 0. \quad [8 \text{ marks}]$$

#### Question 5

(a) A bob of a pendulum swings through an arc of 50cm on its first swing. Each successive swing is 90% of the length of the previous swing. Find the total distance travelled by the bob in coming to rest.    [4 marks]

(b) Prove

$$\frac{\cos A}{1 - \tan A} + \frac{\sin A}{1 - \cot A} = \sin A + \cos A. \quad [8 \text{ marks}]$$

(c)

#### Question 6

(a) Find all roots of

$$2x^3 - x^2 - 8x + 4 = 0. \quad [10 \text{ marks}]$$

(b) Solve for  $x$

(i)  $\left(\frac{5}{3}\right)^{x-1} = \frac{81}{725}$ .    [5 marks]

(ii)  $\log_2(x - 1) = 3 - \log_2(x + 1)$     [5 marks]

### Question 7

(a) Use Cramer's rule to solve the system

$$2x - 3y - 3z = 9$$

$$3y + 2z = 0$$

$$3x - 4y + \quad = 1.$$

[13 marks]

(b) Evaluate

$$\left(1 + i\sqrt{3}\right)^6,$$

and express in the form  $a + ib$ .

[7 marks]

---