UNIVERSITY OF SWAZILAND

FINAL EXAMINATIONS 2007/8

BSc./B.Ed. /B.A.S.S.

TITLE OF PAPER	:	INTRODUCTION TO CALCULUS
COURSE NUMBER	:	M 115
TIME ALLOWED	:	THREE (3) HOURS
INSTRUCTIONS	:	1. THIS PAPER CONSISTS OF SEVEN QUESTIONS.
		2. ANSWER ANY FIVE QUESTIONS
SPECIAL REQUIREMENTS	:	NONE

THIS EXAMINATION PAPER SHOULD NOT BE OPENED UNTIL
PERMISSION HAS BEEN GRANTED BY THE INVIGILATOR.

QUESTION 1

1. (a) Evaluate the following first derivatives $\frac{dy}{dx}$

     i. $y = 3x^4 - x^2 + 2x$

     ii. $x = 3t^2 - 2t, \ y = t^3 - 3t$

     iii. $y = x^{\sin x}$ [12]

   (b) Evaluate the second derivatives $\frac{d^2y}{dx^2}$ of the following

     i. $y = \sin(3x + 2)$

     ii. $xy + y^2 = 1$ [8]

QUESTION 2

2. (a) Evaluate the following indefinite integrals

     i. $\int (x^4 + 3x^2 + \frac{1}{x} + \frac{1}{x^5}) dx$

     ii. $\int \cos^3 x \, dx$

     iii. $\int \frac{1}{\sqrt{\ln x} \ x} dx$ [12]

   (b) Derive the reduction formula in (i) and use it to evaluate the integral in (ii)

     i. $\int \sin^m x \, dx = \frac{-\sin^{m-1} x \cos x}{m} + \frac{m-1}{m} \int \sin^{m-2} x \, dx$

     ii. $\int \sin^5 x \, dx$

QUESTION 3

3. (a) Evaluate the following limits

     i. $\lim_{x \to 2} \frac{x^2 - 4}{x - 2}$

     ii. $\lim_{x \to 0} \frac{\sqrt{1 - x} - \sqrt{1 + x}}{x}$ [8]

   (b) Use the definition (not formulas) to find the derivatives of the following functions

     i. $f(x) = 3x^2 + 5$

     ii. $f(x) = \sqrt{x}$

     iii. $f(x) = x^4$ [12]

QUESTION 4

4. (a) Find the equation of the tangent to the curve

$$y = x^4 - 2x^3 + 3$$

at the point $(-1, 6)$ [5]

(b) Find all relative maxima and minima of the function

$$y = 3x^4 + 4x^3 - 12x^2 + 2$$

[4]

(c) Find the area enclosed between $y = 10 + 3x - x^2$ and $y = 2x + 4$ [8]

QUESTION 5

5. (a) If n is a positive integer, make deductions about the nth derivative $\frac{d^n y}{dx^n}$ for

$$y = \frac{1}{2 - 3x}$$

[6]

(b) Use trig, substitution to evaluate the following

   i. $\int \frac{dx}{x\sqrt{x^2 - 1}}$ [5]

   ii. $\int \frac{x^2}{(9 - x^2)^{\frac{3}{2}}} dx$ [5]

(c) Evaluate the following definite integral $\int_0^2 (16x - 3x^2 + x^3) dx$ [4]

QUESTION 6

6. (a) Show that

$$\int x\sqrt{1+x}\,dx = \frac{2}{15}(1+x)^{\frac{3}{2}}(3x-2) + C$$

   in two ways

      i. using the substitution $u = \sqrt{1+x}$ [5]

      ii. using integration by parts [5]

   (b) Find $\frac{dy}{dx}$ in the following

$$y\cos 2x = x\sin 2y$$

[5]

   (c) Evaluate $\frac{d^2y}{dx^2}$ if

$$y = \cosh(3x^2 + 5)$$

[5]

QUESTION 7

7. (a) Use partial fractions to evaluate the following integrals

      i. $\displaystyle\int \frac{x}{x^2 + 2x - 3}\,dx$ [5]

      ii. $\displaystyle\int \frac{x^4 - x^3 - x - 1}{x^3 - x^2}\,dx$ [7]

   (b) Find $\frac{dy}{dx}$ in the following

      i. $y = arc\cos(\frac{1}{x})$ [4]

      ii. $y^2 = \frac{x}{x+1}$ [4]