

UNIVERSITY OF SWAZILAND

SUPPLEMENTARY EXAMINATIONS 2007/8

BSc. /BEd. /B.A.S.S I

TITLE OF PAPER : CALCULUS

COURSE NUMBER : M 115

TIME ALLOWED : THREE (3) HOURS

INSTRUCTIONS : 1. THIS PAPER CONSISTS OF
SEVEN QUESTIONS.
2. ANSWER ANY FIVE QUESTIONS
3. Calculators may be used.

SPECIAL REQUIREMENTS : NONE

THIS EXAMINATION PAPER SHOULD NOT BE OPENED UNTIL
PERMISSION HAS BEEN GRANTED BY THE INVIGILATOR.

QUESTION 1

1. (a) (i) Use the definition of the derivative to find $f'(x)$ given that
 $f(x) = \frac{1}{x}$. [5 marks]
- (ii) Evaluate $\int e^{\cos 3x} \sin 3x dx$. [5 marks]
- (b) Evaluate the following limits
- (i) $\lim_{x \rightarrow 1} \frac{x^3 - 4x^2 + 2x + 1}{x^2 + x - 2}$. [5 marks]
- (ii) $\lim_{x \rightarrow \frac{\pi}{2}} \frac{\cos x}{\frac{\pi}{2} - x}$. [5 marks]

QUESTION 2

2. (a) (i) Find the general expression for $\frac{d^n y}{dx^n}$ when $y = (x + 2)^{\frac{1}{3}}$.
(Here $n \geq 1$ is a positive integer.) [6 marks]
- (ii) Use logarithmic differentiation to find $f'(x)$ for
 $f(x) = \frac{x(x - 5)^{\frac{1}{3}}}{1 + \sin^3 x}$. [7 marks]
- (b) Integrate $\int \cos^3 x \sin^4 x dx$. [7 marks]

QUESTION 3

3. (a) Derive the following reduction formula

$$\int \cos^n x dx = \frac{1}{n} \cos^{n-1} x \sin x + \frac{n-1}{n} \int \cos^{n-2} x dx$$

and use it to evaluate $\int \cos^5 x dx$. [12 marks]

- (b) Evaluate the indefinite integral

$$\int \frac{2x + 4}{x^3 - 4x} dx$$

[8 marks]

QUESTION 4

4. (a) (i) Find the equation of the line tangent to the curve $x^4 - y^4 - 4xy = 7$ at the point $(2, 1)$. [5 marks]
(ii) Evaluate the indefinite integral

$$\int \frac{\sin x}{1 + \cos x} dx$$

[5 marks]

- (b) Find $\frac{dy}{dx}$ when

(i) $y = \ln \left(\frac{5^x}{\cos x} \right)$ (ii) $y = \tan(e^{-2x})$ [10 marks]

QUESTION 5

5. (a) Evaluate:

(i) $\int \frac{dx}{\sqrt{8 - 2x - x^2}}$ (ii) $\int \sin 3x \sin x dx$. [10 marks]

- (b) Show that the function

$$y = a \sin kx + b \cos kx$$

where a, b and k are constants, is a solution of the equation

$$y'' + k^2 y = 0$$

[10 marks]

QUESTION 6

6. (a) Integrate the following:

(i) $\int x^2 \ln(\sqrt{x}) dx$ (ii) $\int \frac{\sin 2x}{\sin x + 3} dx$. [10 marks]

- (b) (i) Given that $x = 3(\cos t - t \sin t)$ and $y = 3(\sin t + t \cos t)$, find

$\frac{dy}{dx}$ (in terms of t). [5 marks]

- (ii) Evaluate

$\lim_{x \rightarrow \infty} \frac{x^2 + 3}{\sqrt{5x^4 - x}}$. [5 marks]

QUESTION 7

7. (a) (i) Find the area between the curves $y - x = 0$ and $y = x^2$. [5 marks]
(ii) Use Leibnitz's rule to find $y^{(4)}$ for $y = e^{-2x} \sin 4x$ [5 marks]
- (b) Find $\frac{dy}{dx}$ in each of the following
- (i) $y = (\sin x^2)^{2x}$. [5 marks]
(ii) $y = x^{\frac{1}{2}} - (x^2 + 4) \tan\left(\frac{x^2}{2}\right)$. [5 marks]