UNIVERSITY OF SWAZILAND

FINAL EXAMINATIONS 2007

BSc. / BEd. / B.A.S.S. II

TITLE OF PAPER	:	CALCULUS 1
COURSE NUMBER	:	M 211
TIME ALLOWED	:	THREE (3) HOURS
INSTRUCTIONS	:	1. THIS PAPER CONSISTS OF SEVEN QUESTIONS. 2. ANSWER ANY FIVE QUESTIONS 3. ONLY NON-PROGRAMMABLE CALCULATORS MAY BE USED.
SPECIAL REQUIREMENTS	:	NONE

THIS EXAMINATION PAPER SHOULD NOT BE OPENED UNTIL PERMISSION HAS BEEN GRANTED BY THE INVIGILATOR.

QUESTION 1

(a) State and prove the Mean Value Theorem (MVT). [10 marks]

(b) Find the value or values of c that satisfy the equation given by the MVT for the function below on the interval given

$$f(x) = \ln(x-1) \quad [2,4]$$

[5 marks]

(c) Does the function $f(x) = x^{\frac{2}{3}}$ on the interval $[-1,8]$ satisfy the hypothesis of the MVT? Give reasons for your answer. [5 marks]

QUESTION 2

(a) Define the terms local (relative) maximum and local (relative) minimum. [6 marks]

(b)State (without proof) the First Derivative Theorem for local extreme values. [2 marks]

(c)Use the above theorem to find the local extreme values of the following functions.

(i) $f(x) = x^2 - 1 \quad [-1,2]$ [6 marks]

(ii) $f(\theta) = \sin\theta \quad [\frac{-\pi}{2}, \frac{5\pi}{6}]$ [6 marks]

QUESTION 3

(a) Give any three (3) assumptions that are made about the functions $f(x)$ and/or $g(x)$ in an interval $[a, b]$ for the L'Hopital Rule

$$\lim_{x \to c} \frac{f(x)}{g(x)} = \lim_{x \to c} \frac{f'(x)}{g'(x)}$$

to hold, provided $\lim_{x \to c} \frac{f'(x)}{g'(x)}$ exists. [6 marks]

(b) Evaluate the limit of the following functions if they exist:

(i) $\lim_{x \to 0} \frac{1 - \cos(x)}{x + x^2}$ [4marks]

(ii) $\lim_{x \to \infty} \frac{e^x}{x^2}$ [5marks]

(iii) $\lim_{x \to \infty} x^{\frac{1}{x}}$ [5marks]

QUESTION 4

(a)Derive the **shell formula** for finding the volumes of solids of revolution about a vertical line L. [10 marks]

(b) Use the shell formula above to find the volume of the solid generated by revolving the region bounded by the curve $y = x^2$,and the lines $y = 2 - x$ and $x = 0$ for $x \geq 0$ about the y-axis. [5 marks]

(c) Use the **washer method** to find the volume of the same solid above. [5marks]

QUESTION 5

(a) Show that the length of the curve

$x = r\cos^3\theta \qquad y = r\sin^3\theta \quad \text{for } 0 \leq \theta \leq 2\pi \quad \text{is } 6r.$ [5 marks]

(b)Use the disc method to find the volume of the solid generated by revolving the region bounded by the curve $y = \sqrt{9 - x^2}$ and the line $y = 0$

about the line $y = 0$. [5 marks]

(c) For the following sequences, determine whether they converge or diverge. If the sequence converges, find its limit as $n \to \infty$.

(i) $a_n = \frac{\sin(n)}{n^2}$ [5 marks]]

(ii) $a_n = (1 - \frac{1}{n})^n$ [5 marks]

QUESTION 6

(a) A sequence a_n is said to be monotone increasing if (i)—, and monotone decreasing if (ii)—. [4 marks]

(b) Suppose that $f(x)$ is defined for all $0 \leq x \leq 1$, that f is differentiable at $x = 0$, and that $f(0) = 0$. Define a sequence $a_n = nf(\frac{1}{n})$.

(i) Show that $\lim a_n = f'(0)$. [10 marks]

(ii) Use the result in (i) to find the limit of the sequence

$a_n = n\tan^{-1}\frac{1}{n}$ [6 marks]

QUESTION 7

(a) State an appropriate test, to test for convergence or divergence of the following series;

(i) $\sum_{n=1}^{\infty} \frac{2n}{3n-1}$ [5marks]

(ii) $\sum_{n=2}^{\infty} \frac{\ln n}{n}$ [5marks]

(iii) $\sum_{n=1}^{\infty} \frac{n^{10}}{10^n}$ [5marks]

(b) For the following series determine whether it diverges, converges, or converges absolutely.

$\sum_{n=1}^{\infty} (-1)^{n+1} \frac{n}{n^3+1}$ [5marks]

End of Paper