UNIVERSITY OF SWAZILAND

SUPPLEMENTARY EXAMINATIONS 2007/8

BSc. / BEd. / B.A.S.S. II

TITLE OF PAPER	:	CALCULUS 1
COURSE NUMBER	:	M 211
TIME ALLOWED	:	THREE (3) HOURS
INSTRUCTIONS	:	1. THIS PAPER CONSISTS OF SEVEN QUESTIONS. 2. ANSWER ANY FIVE QUESTIONS 3. ONLY NON-PROGRAMMABLE CALCULATORS MAY BE USED.
SPECIAL REQUIREMENTS	:	NONE

THIS EXAMINATION PAPER SHOULD NOT BE OPENED UNTIL PERMISSION HAS BEEN GRANTED BY THE INVIGILATOR.

QUESTION 1

(a) Let $f(x) = x + \frac{1}{x}$. Show that the function has a local minimum at $x = -1$ and a local maximum at $x = 1$. [5 marks]

(b) State and prove the Mean Value Theorem (MVT). [10 marks]

(c) Find the value or values of c that satisfy the equation given by the MVT for the function and interval

$f(x) = \sin^{-1} x, \qquad [-1, 1]$ [5 marks]

QUESTION 2

Evaluate the limit of the following functions using L'Hopital's rule.

(a)

$$\lim_{x \to \frac{\pi}{2}} \frac{1 - \sin x}{1 + \cos(2x)}$$

(b)

$$\lim_{x \to \infty} (1 - \frac{1}{x^2})^{x^2}$$

(c)

$$\lim_{x \to 0} x^2 \cot x$$

[6,8,6 marks]

QUESTION 3

(a) Derive the **shell formula** for finding the volumes of solids of revolution about a vertical line L. [10 marks]

(b) Use the shell formula above to find the volume of the solid generated by revolving the region bounded by the curve $y = \sqrt{x}$ and the lines $y = 0$ and $x = 4$ about the x-axis. [10 marks]

QUESTION 4

Suppose that $f(x)$ is defined for all $0 \leq x \leq 1$, that f is differentiable at $x = 0$, and that $f(0) = 0$. Define a sequence $a_n = nf(\frac{1}{n})$.

(a) Show that $\lim_{x\to\infty} a_n = f'(0)$. [10 marks]

(b) Use the result in (a) to find the limit of the following sequences

(i) $a_n = n \tan^{-1} \frac{1}{n}$ [6 marks]

(ii) $a_n = n(e^{\frac{1}{n}} - 1)$ [4 marks]

QUESTION 5

(a)Find the length of the curve given by the following parametric equation

$x = t - \sin t, \qquad y = 1 - \cos t; \qquad 0 \leq t \leq 2\pi.$ [10 marks]

(b) Show that the given sequence is monotone increasing or decreasing,

(i) $a_n = \frac{n}{2^{n+2}}$ (ii) $a_n = \frac{e^{2\sqrt{n}}}{n}$ [10 marks]

QUESTION 6

(a) State the ratio test for infinite series. [4 marks]

(b) Apply the ratio test to the series $\sum_{k=1}^{\infty} \frac{3^k}{k!.k}$ to determine whether the series converges or diverges. [6 marks].

(c) Prove that the harmonic series $\sum_{n=1}^{\infty} \frac{1}{n}$ diverges. [10 marks]]

QUESTION 7

(a) Use the **disc method** to find the volume of the solid generated by the region bounded by the curve $y = \sqrt{x}$ and the lines $y = 1$ and $x = 4$ about the line $y = 1$. [10 marks]

(b) Find, using the **washer method,** the volume of the solid generated by revolving the region bounded by the curve $y = x^2$, and the lines $y = 2 - x$ and $x = 0$ for $x \geq 0$ about the y-axis. [10 marks]

End of Paper