UNIVERSITY OF SWAZILAND

FINAL EXAMINATIONS 2008

BSc. / BEd. / B.A.S.S. II

TITLE OF PAPER	:	CALCULUS II
COURSE NUMBER	:	M 212
TIME ALLOWED	:	THREE (3) HOURS
INSTRUCTIONS	:	1. THIS PAPER CONSISTS OF SEVEN QUESTIONS. 2. ANSWER ANY FIVE QUESTIONS 3. ONLY NON-PROGRAMMABLE CALCULATORS MAY BE USED.
SPECIAL REQUIREMENTS	:	NONE

THIS EXAMINATION PAPER SHOULD NOT BE OPENED UNTIL PERMISSION HAS BEEN GRANTED BY THE INVIGILATOR.

QUESTION 1

(a) Find the Taylor Series generated by the function $f(x) = \cos x$ at $x = 0$. [8 marks]

(b) From the formula derived for the Taylor Series of $\cos x$ in (a) above, derive the Maclaurin series for $\cos x$. [4 marks]

(c) Use the Binomial series to estimate $\sqrt{1.25}$ with an error of less than 0.001. [8 marks]

QUESTION 2

(a) Find the area of the region that lies inside the circle $r = 1$ and outside the cardioid $r = 1 - \cos\theta$. [5 marks]

(b) Replace the following polar equation by the equivalent cartesian equations and identify its graph, $r^2 = 4r\cos\theta$. [7 marks]

(c) Find all the polar coordinates of the point $P(2, \frac{\pi}{6})$. [8 marks]

QUESTION 3

(a)If $f(x, y) = x\cos y + ye^x$, find the second-order derivatives:
$\frac{\partial^2 f}{\partial y^2}$ and $\frac{\partial^2 f}{\partial y \partial x}$ [5 marks]

(b) Show that the equation $x^2 - 4y^2 + 2x + 8y - 7 = 0$ represents a hyperbola. Find its center, asymptotes, and foci. [7 marks]

(c) Find the directrix and the eccentricity of the parabola $r = \frac{25}{10+10\cos\theta}$ [5 marks]

(d) Find the polar equation for a conic with $e = 1$ and $y = -6$. [3 marks]

QUESTION 4

(a) Find $\lim_{(x,y)\to(0,0)} \dfrac{x^2 - xy}{\sqrt{x} - \sqrt{y}}$ [6 marks]

(b) Show that $f(x,y) = \begin{cases} \frac{2xy}{x^2+y^2} & (x,y) \neq (0,0) \\ 0 & (x,y) = (0,0) \end{cases}$ is continuous at every point except the origin. [10 marks]

(c) Find $\frac{\partial z}{\partial x}$ if the equation $yz - \ln z = x + y$ defines z as a function of the two independent variables x and y and the partial derivative exists. [4 marks]

QUESTION 5

(a) Show that $f_{xy} = f_{yx}$ for the function $f = xy + \frac{e^y}{y^2+1}$. [6 marks]

(b) Use the chain rule to find the derivative of $w = xy$ with respect to t along the path $x = \cos t, y = \sin t$. [7 marks]

(c) Find a spherical coordinate equation for the sphere $x^2 + y^2 + (z-1)^2 = 1$. [7 marks]

QUESTION 6

(a) Evaluate the iterated integral $\int_0^3 \int_0^2 (4-y^2)dydx$. [4 marks]

(b) Express $\frac{\partial w}{\partial r}$ and $\frac{\partial w}{\partial s}$ in terms of r and s if $w = x^2 + y^2$,

$x = r - s, \quad y = r + s.$ [6 marks]

(c) Evaluate the volume of an 'ice cream cone' cut from a solid sphere $\rho \leq 1$ by the cone $\phi = \frac{\pi}{3}$ given by

$$V = \int_0^{2\pi} \int_0^{\frac{\pi}{3}} \int_0^1 \rho^2 \sin\phi \; d\rho \; d\phi \; d\theta.$$

[6 marks]

(d) Find a spherical coordinate equation for the cone $z = \sqrt{x^2+y^2}$. [4 marks]

QUESTION 7

(a) Find a polar equation for the circle $x^2 + (y-3)^2 = 9$. [5 marks]

(b) Find $\frac{\partial f}{\partial y}$ as a function if $f(x,y) = y \sin xy$ [5 marks]

(c) Suppose that we substitute polar coordinates $x = r\cos\theta$ and $y = r\sin\theta$ in a differentiable function $w = f(x,y)$. Show that

(i) $\frac{\partial w}{\partial r} = f_x \cos\theta + f_y \sin\theta$ and

(ii) $\frac{1}{r}\frac{\partial w}{\partial \theta} = -f_x \sin\theta + f_y \cos\theta$ [3,3 marks]

(d) Evaluate the double integral over the region R.

$\iint_R (6y^2 - 2x)dA \quad R: \; 0 \leq x \leq 1, \;\; 0 \leq y \leq 2$ [4 marks]