
University of Swaziland



Final Examination, April/May 2008

BSc II, Bass II, BEd II

Title of Paper : Ordinary Differential Equations

Course Number : M213

Time Allowed : Three (3) hours

Instructions :

1. This paper consists of SEVEN questions.
2. Each question is worth 20%.
3. Answer ANY FIVE questions.
4. Show all your working.

THIS PAPER SHOULD NOT BE OPENED UNTIL PERMISSION HAS BEEN GIVEN BY THE INVIGILATOR.

Question 1

(a) Find the general solution of

$$xydy = (1 + y)(1 - x)dx. \quad [8 \text{ marks}]$$

(b) Find a solution of the equation

$$y'' - 6y' + 9y = e^{3x}$$

using the method of

(i) variation of parameters [6 marks]

(ii) undetermined coefficients. [6 marks]

Question 2

(a) Obtain the general solution of

$$y^{iv} + 3y'' - 4y = 0. \quad [8 \text{ marks}]$$

(b) Use the method of Laplace transforms to solve

$$y'' + 4y = 0, \quad y\left(\frac{\pi}{8}\right) = \sqrt{2}, \quad y'\left(\frac{\pi}{8}\right) = -4\sqrt{2}. \quad [12 \text{ marks}]$$

Question 3

Use the method of Frobenius to find a series solution of

$$x(x - 1)y'' + (3x - 1)y' + y = 0$$

about $x = 0$. [20 marks]

Question 4

- (a) Your friends have an argument. One says “All exact ODEs are separable” while the other claims that “All separable ODEs are exact.” Now they have come to you for clarification. Is any one of them correct? If so, which one? Discuss.

[7 marks]

- (b) Find the general solution of

$$y'' + y' - 6y = \sin 2x. \quad [13 \text{ marks}]$$

Question 5

- (a) Solve

$$y^2 dx + (3xy - x^2) dy = 0, \quad y(1) = 1. \quad [12 \text{ marks}]$$

- (b) Find the general solution of

$$\left(\frac{2x}{y} - \frac{y}{x^2}\right) dx + \left(\frac{1}{x} - \frac{x^2}{y^2}\right) dy = 0. \quad [8 \text{ marks}]$$

Question 6

- (a) Find the general solution of

$$9y'' - 6y' + 4y = 12. \quad [6 \text{ marks}]$$

- (b) Find the solution of

$$y'' - 2y' + 5y = -2x$$

that satisfies the conditions $y(0) = 0$, $y'(0) = -\frac{2}{5}$.

[14 marks]

Question 7

(a) Solve

$$(x + y)dy + (3x + 3y - 4)dx = 0. \quad [15 \text{ marks}]$$

(b) Obtain the general solution of

$$2x^2y'' - 3xy' - 3y = 0. \quad [5 \text{ marks}]$$
