University of Swaziland

Supplementary Examination 2008

BSc II, Bass II, BEd II

Title of Paper : Ordinary Differential Equations

Course Number : M213

Time Allowed : Three (3) hours

Instructions :

1. This paper consists of SEVEN questions.
2. Each question is worth 20%.
3. Answer ANY FIVE questionss
4. Show all your working.

THIS PAPER SHOULD NOT BE OPENED UNTIL PERMISSION HAS BEEN GIVEN BY THE INVIGILATOR.

Question 1

(a) Find the general solution of

$$xe^{2xy}\mathrm{d}x = (1+y)\mathrm{d}y. \qquad \text{[10 marks]}$$

(b) Find a solution of the equation

$$y'' - 7y' + 12y = x^2 \qquad \text{[10 marks]}$$

Question 2

(a) Obtain the general solution of

$$y^{iv} - 16y = 0. \qquad \text{[10 marks]}$$

(b) Use the method of Laplace transforms to solve

$$\ddot{y} + 4\dot{y} + 4y = e^{-2t}, \quad y(0) = 0, \; y'(0) = 0. \quad \text{[10 marks]}$$

Question 3 Consider the equation

$$y' - 2xy = 0. \qquad (1)$$

(a) Find the general solution of (1) analytically. [6 marks]

(b) Find a series solution of (1) about $x = 0$. [14 marks]

Question 4

(a) Your friend claims that

"All linear first order ODEs can be made exact."

Is the statement right or wrong? Discuss.

[7 marks]

(b) Find the general solution of

$$y'' + 2y' + 10y = e^{-x}. \qquad \text{[13 marks]}$$

Question 5

(a) Solve

$$y\big(\ln y - \ln x - 1\big)\mathrm{d}x + x\mathrm{d}y = 0, \quad y(1) = 1. \quad \text{[12 marks]}$$

(b) Find the general solution of

$$\frac{1}{x^2 y}\mathrm{d}x + \left(\frac{1}{xy^2} - 2y\right)\mathrm{d}y = 0. \qquad \text{[8 marks]}$$

Question 6

(a) Find the general solution of

$$y'' - 16y = 0. \qquad \text{[6 marks]}$$

(b) Find the solution of

$$y'' - 6y' + 9y = 0$$

that satisfies the conditions $y(0) = y'(0) = 2$.

[14 marks]

Question 7

(a) Solve

$$yy' - xy^2 + x^2 = 0. \qquad \text{[10 marks]}$$

(b) Using the *method of variation of parameters*, obtain the general solution of

$$y'' - 3y' + 2y = e^{3x}. \qquad \text{[10 marks]}$$

Table of Laplace Transforms

$f(t)$	$F(s)$
t^n	$\frac{n!}{s^{n+1}}$
$\frac{1}{\sqrt{t}}$	$\sqrt{\frac{\pi}{s}}$
e^{at}	$\frac{1}{s-a}$
$t^n e^{at}$	$\frac{n!}{(s-a)^{n+1}}$
$\frac{1}{a-b}(e^{at}-e^{bt})$	$\frac{1}{(s-a)(s-b)}$
$\frac{1}{a-b}(ae^{at}-be^{bt})$	$\frac{s}{(s-a)(s-b)}$
$\sin(at)$	$\frac{a}{s^2+a^2}$
$\cos(at)$	$\frac{s}{s^2+a^2}$
$\sin(at)-at\cos(at)$	$\frac{2a^3}{(s^2+a^2)^2}$
$e^{at}\sin(bt)$	$\frac{b}{(s-a)^2+b^2}$
$e^{at}\cos(bt)$	$\frac{s-a}{(s-a)^2+b^2}$
$\sinh(at)$	$\frac{a}{s^2-a^2}$
$\cosh(at)$	$\frac{s}{s^2-a^2}$
$\sin(at)\sinh(at)$	$\frac{2a^2}{s^4+4a^4}$
$\sinh(at)\sin(at)$	$\frac{2a^3}{s^4-a^4}$
$f^{(n)}(t)$	$s^nF(s)-s^{n-1}f(0)-\ldots-f^{n-1}(0)$