UNIVERSITY OF SWAZILAND

FINAL EXAMINATIONS 2007/8

BSc. II

TITLE OF PAPER	:	MATHEMATICS FOR SCIENTISTS
COURSE NUMBER	:	M 215
TIME ALLOWED	:	THREE (3) HOURS
INSTRUCTIONS	:	1. THIS PAPER CONSISTS OF SEVEN QUESTIONS. 2. ANSWER ANY FIVE QUESTIONS
SPECIAL REQUIREMENTS	:	NONE

THIS EXAMINATION PAPER SHOULD NOT BE OPENED UNTIL PERMISSION HAS BEEN GRANTED BY THE INVIGILATOR.

QUESTION 1

1. (a) Given that $f(x,y) = \dfrac{2y}{y+\cos x}$, find f_x and f_y [4 Marks]

   (b) Find

      i. $\lim_{x\to 0}(1+x)^{\frac{1}{x}}$ [3 Marks]

      ii. $\lim_{t\to\infty} \dfrac{t^2+t}{2t^2+1}$ [3 Marks]

   (c) Locate all relative extrema and saddle points of $f(x,y) = x^3 + y^3 - 2xy + 6$. Find the function value at these points. [10 Marks]

QUESTION 2

2. (a) If

$$\begin{aligned} \underline{a} &= i - j + 2k \\ \underline{b} &= (2,1,1) \\ \underline{c} &= i + 2j - k, \end{aligned}$$

   show that $(\underline{a} \times \underline{b}) \times \underline{c} = (\underline{a}\cdot\underline{c})\underline{b} - (\underline{b}\cdot\underline{c})\underline{a}$ [5 Marks]

   (b) Find the first five terms of the Taylor series generated by $f(x) = \frac{1}{x}$ at $x = 2$ [6 Marks]

   (c) Find the first five terms of the Mcclaurins series generated by $f(x) = (1+x)^3$. Use the series to evaluate $\int_1^2 (1+x)^3 dx$ [9 Marks]

QUESTION 3

3. (a) Use the method of Lagrange to find the greatest and the smallest values of $f(x,y) = xy$ subject to $g(x,y) = \dfrac{x^2}{8} + \dfrac{y^2}{2} = 1$ [10 Marks]

   (b) Solve the differential equation $(x^2+y^2)dx + (2xy + \cos y)dy = 0$ [7 Marks]

   (c) For the function $f(x) = \sqrt{x-1}$ find c in $1 \le x \le 3$ that satisfies the mean value theorem. [3 Marks]

QUESTION 4

4. (a) Use polar coordinates to evaluate the integral $\int\int_R \frac{dx\,dy}{1+x^2+y^2}$, where R is the region in the first quadrant bounded by $y = 0$, $y = x$ and $x^2 + y^2 = 4$ [10 Marks]

   (b) Solve the differential equation $(x^3 + y^3)dx + xy^2\ dy = 0$ [10 Marks]

QUESTION 5

5. (a) Sketch and evaluate the area of the region enclosed by $\int_0^4 \int_{\frac{y}{2}}^{y} (4x + 2)dx\ dy$ [10 Marks]

   (b) Reverse the order and evaluate the resulting integral. [5 Marks]

   (c) Find the spherical coordinates equation of $x^2 + y^2 + (z-1)^2 = 1$ [5 Marks]

QUESTION 6

6. (a) Use the chain rule to express $\frac{\partial w}{\partial s}$ and $\frac{\partial w}{\partial r}$ and evaluate at $(r, s) = (\frac{1}{2}, 1)$ if

$$\begin{aligned} w &= x^2 + 2y + Z^2 \\ x &= \frac{r}{s}, y = r^2 + \ln s \\ Z &= 2r \end{aligned}$$

[10 Marks]

(b) If $u = u(x, y)$, $x = r\cos\theta$, $y = r\sin\theta$, Show that
$\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial u}{\partial y}\right)^2 = \left(\frac{\partial u}{\partial r}\right)^2 + \frac{1}{r^2}\left(\frac{\partial u}{\partial \theta}\right)^2$ [6 Marks]

(c) Use differentials to approximate $(64.08)^{\frac{1}{3}}$ [4 Marks]

QUESTION 7

7. (a) Evaluate the iterated integral $\int_0^1 \int_0^1 \frac{y}{(xy+1)^2} dx\ dy$ [10 Marks]

(b) Use implicit differentiation to find y'

$$x^3 + (x - y)^2 = (x + y)^2 - y^3$$

[5 Marks]

(c) If $Z = Z(x, y)$ and

$$\begin{aligned} x &= e^{r+s} + e^{r-s} \\ y &= e^{r+s} - e^{r-s} \end{aligned}$$

find Z_{rr} [6 Marks]