UNIVERSITY OF SWAZILAND

SUPPLEMENTARY EXAMINATIONS 2007/8

BSc. II

TITLE OF PAPER	:	MATHEMATICS FOR SCIENTISTS
COURSE NUMBER	:	M 215
TIME ALLOWED	:	THREE (3) HOURS
INSTRUCTIONS	:	1. THIS PAPER CONSISTS OF SEVEN QUESTIONS. 2. ANSWER ANY FIVE QUESTIONS
SPECIAL REQUIREMENTS	:	NONE

THIS EXAMINATION PAPER SHOULD NOT BE OPENED UNTIL
PERMISSION HAS BEEN GRANTED BY THE INVIGILATOR.

QUESTION 1

1. (a) Use the chain rule to find $\frac{\partial f}{\partial s}$ and $\frac{\partial f}{\partial r}$ at $(r,s) = \quad (\frac{1}{2},3)$ if $f(x,y) = x^2 + y^2$ and $x = r - s, \quad y = r + s$ [10 Marks]

   (b) Evaluate $\int\int\int_R \quad s^2y^2Z \quad dx\ dy\ dz$ where $R = \{0 \leq z \leq 3, \quad 0 \leq x \leq 1, \quad 2 \leq y \leq 3\}$ [5 Marks]

   (c) Solve the differential equation $\frac{d^2y}{dx^2} + \frac{4dy}{dx} + 4y = 0$ [5 Marks]

QUESTION 2

2. (a) Find the area of the triangle with vertices $P(1,-1,0) \quad Q(2,1,-1) \quad R(-1,1,2)$ [5 Marks]

   (b) Evaluate the iterated integral $\int_0^x \int_{\frac{\pi}{6}}^{\pi} \quad \frac{\sin x}{x} dy \ \ dx$ [5 Marks]

   (c) Evaluate

   i. $\lim_{x \to 0} \left(\frac{1}{x} - \frac{1}{ln(1+x)} \right)$ [5 Marks]

   ii. $\lim_{x \to \infty} \frac{\frac{1}{x}}{\sin \frac{\pi}{x}}$ [5 Marks]

QUESTION 3

3. (a) Find the first 4 non zero terms of the Mcclairins series of $f(x) = \sin x$. Use the series to evaluste $\int_0^1 \sin x^2 dx$ [10 Marks]

   (b) Locate all relative extrema and saddle points of $f(x,y) = 3x^2y^2 + 2y^3 + 6x^2 - 12y$ [10 Marks]

QUESTION 4

4. (a) Use the method of Lagrange multipliers to find the maximum and the minimum values of $f(x, y) = 3x + 4y$ subject to the constraints $x^2 + y^2 = 1$ [10 Marks]

   (b) Solve the differential equation $(3x^2 + 2xy^2 + 4y)dx + (2x^2y + 4x + 5y^4)dy = 0$ [10 Marks]

QUESTION 5

5. (a) Use polar coordinates to evaluate the area enclosed by $x^2 + y^2 = 4$ and $y^2 + x^2 = 9$ if $\int\int_R (x^2 + y^2)dy \ dx$ [10 Marks]

   (b) Solve the differential equation $(x^3 + y^3)dx + xy^2dy = 0$. [10 Marks]

QUESTION 6

6. (a) Use differentials to find the approximate value of $\sqrt{35.6} \cdot 3\sqrt{64.08}$ [5 Marks]

(b) Solve the differential equation $y''' - y'' - 10y' - 8y = 0$ [6 Marks]

(c) If $U = U(x, y)$ and

$$x = r\cos\theta$$
$$y = r\sin\theta$$

$$\frac{\partial^2 U}{\partial x^2} + \frac{\partial^2 U}{\partial y^2} = \frac{\partial^2 U}{\partial r^2} + \frac{1}{r}\frac{\partial^2 U}{\partial \theta^2}$$ [9 Marks]

QUESTION 7

7. (a) Sketch and evaluate the area bounded by $f(x) = \sin x$ and $g(x) = \cos x$ between $x = \frac{5\pi}{4}$ and $x = \frac{\pi}{4}$ [8 Marks]

(b) If $Z = xy \in^{\frac{x}{y}}$ to $x = r\cos\theta$ and $Z = r\sin\theta$ find $\frac{dz}{d\theta}$ when $\theta = \frac{\pi}{6}$ and $r = 2$. [7 Marks]

(c) Verify that for $f(x) = \sqrt{x-1}$ in $[1, 2]$ the mean value theorem is satisfied. Find $d \in (1, 2)$ whose existence is guaranteed. [5 Marks]