UNIVERSITY OF SWAZILAND

FINAL EXAMINATIONS 2007/8

BSc. /B.Ed. /B.A.S.S

TITLE OF PAPER	:	LINEAR ALGEBRA
COURSE NUMBER	:	M 220
TIME ALLOWED	:	THREE (3) HOURS
INSTRUCTIONS	:	1. THIS PAPER CONSISTS OF SEVEN QUESTIONS. 2. ANSWER ANY FIVE QUESTIONS
SPECIAL REQUIREMENTS	:	NONE

THIS EXAMINATION PAPER SHOULD NOT BE OPENED UNTIL PERMISSION HAS BEEN GRANTED BY THE INVIGILATOR.

QUESTION 1

1. (a) Verify the Cayle-Hamilton theorem for the following matrix

$$A = \begin{pmatrix} 1 & 1 \\ 3 & -2 \end{pmatrix}$$

[8]

(b) Find the eigenvalues and eigenvectors of the matrix

$$A = \begin{pmatrix} 1 & 2 & -1 \\ 1 & 0 & 1 \\ 4 & -4 & 5 \end{pmatrix}.$$

[8]

(c) Determine whether the following has a non-trivial solution

$$\begin{aligned} 2x_1 + x_2 - x_3 + 2x_4 &= 0 \\ x_1 + x_2 + x_3 + x_4 &= 0 \\ 3x_1 + 2x_2 + 2x_3 + 2x_4 &= 0 \end{aligned}$$

[4]

QUESTION 2

2. (a) Prove that if A and B are both non-singular $n \times n$ matrices, then AB is a non-singular and $(AB)^{-1} = B^{-1}A^{-1}$ [5]

(b) Find standard matrices for the following liner transformations

i. $T : \mathbb{R}^3 \to \mathbb{R}^2 : T(x, y, z) = (x + z, \quad y - z)$ [5]

ii. $T : \mathbb{R}^3 \to \mathbb{R}^4 : T(x, y, z) = (x, y, z, x + z)$ [5]

(c) Let $B^1 = \{v_1, v_2, v_3\}$ and $B = \{u_1, u_2, u_3\}$ be bases in $\mathbb{R}^3$ where $v_1 = (0, 2, 1)$ $v_2 = (1, 0, 2)$ $v_3 = (1, -1, 0)$ $u_1 = (1, 0, 0)$ $u_2 = (1, 1, 0)$ $u_3(1, 1, 1)$

Find the transition matrix from B^1 to B [5]

QUESTION 3

3. (a) Prove that if a homogeneous system has more unknown than the number of equations then it always has non-trivial solutions [10]

(b) Use Gaussian elimination to solve the following

$$\begin{aligned} x - y + z &= 7 \\ 2x - y &= 9 \\ -x + y + z &= 2 \end{aligned}$$

[5]

(c) Prove that the set

$$B = \{x^2 + 1, \quad x - 1, \quad 2x + 2\}$$

is a basis for the vector space $P_2(x)$ [5]

QUESTION 4

4. (a) Show that the matrix A is non singular by computing A^{-1}

$$A = \begin{pmatrix} 1 & 1 & 1 \\ 0 & 2 & 3 \\ 3 & 5 & 2 \end{pmatrix}$$

[5]

(b) Use the inverse A^{-1} in (a) to solve the system $Av = b$ where $v = (x_1, x_2, x_3)$ and $b = (1, -1, 3)$ [5]

(c) For the matrix $A = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 2 & 1 \\ 2 & 3 & 2 \end{pmatrix}$ for the augmented matrix $(A : I)$ and hence find the inverse A^{-1} [5]

(d) Find a finite sequence of elementary matrices $E_1, E_2, \cdots, E_h$ such that $E_h E_{h-1}, \cdots E_1 A = I$ in (c) above. [5]

QUESTION 5

5. (a) Let $v_1 = (1, 0, 3)$ $v_2 = (5, 2, 1)$ and $v_3 = (0, 1, 6)$. Determine whether the set $B = \{v_1, v_2, v_3\}$ form a linearly depended or linearly independent set over $\mathbb{R}$ [6]

(b) $B = \{(1, 1, 1), \quad (-1, 1, 0), \quad (1, 0, -1)\}$, is an $\mathbb{R}$ basis for $\mathbb{R}^3$. Find the co-ordinates of

i. (1,0,0) [4]

ii. (1,2,3) [4]

(c) Show that the set $B = \{v_1, v_2, v_3, v_4\}$ where $v_1 = (1, 0, 1, 0)$, $v_2 = (0, 1, -1, 2)$, $v_3 = (0, 2, 2, 1)$ and $v_4 = (1, 0, 0, 1)$ is a basis for $\mathbb{R}^4$. [6]

QUESTION 6

6. (a) Use Cramer's rule to solve:

$$\begin{aligned} 2x_1 + x_2 - x_3 &= 0 \\ x_1 - x_2 + 3x_3 &= 1 \\ 2x_1 + 2x_2 + x_3 &= 7 \end{aligned}$$

[5]

(b) Use Gaussian elimination to solve

$$\begin{aligned} x + y + 2z + 3w &= 13 \\ x - 2y + z + w &= 8 \\ 3x + y + z - w &= 1 \end{aligned}$$

[5]

(c) For any square matrix A, show that $A - A^T$ is a skew-symmetric matrix [5]

(d) For any square matrix A, show that $A + A^T$ is a symmetric matrix [5]

QUESTION 7

7. (a) Determine whether the following mappings are linear transformations

i. $T : \mathbb{R}^3 \rightarrow \mathbb{R}^2$ defined by $T(x, y, z) = (x + y - z, 2x + y)$ [5]

ii. $T : \mathbb{R}^3 \rightarrow \mathbb{R}^2$ defined by $T(x, y, z) = (x + 1, y)$ [5]

(b) Let $T : \mathbb{R}^3 \rightarrow \mathbb{R}^2$ defined by $T(x, y, z) = (x + y + z, \quad x + 2y + 3z)$

i. Find the standard matrix for the transformtion T.

ii. Find the matrix of T relative to the bases

$$B = \{1, 1, 0), \quad (0, 1, 1), \quad (0, 0, 1)\}$$

$$\{(1, 2), \quad (13)\}$$