UNIVERSITY OF SWAZILAND

FINAL EXAMINATIONS 2007/2008

B.Sc. / B.Ed. / B.A.S.S. II

TITLE OF PAPER	:	DYNAMICS I
COURSE NUMBER	:	M255
TIME ALLOWED	:	THREE (3) HOURS
INSTRUCTIONS	:	1. THIS PAPER CONSISTS OF SEVEN QUESTIONS. 2. ANSWER ANY FIVE QUESTIONS
SPECIAL REQUIREMENTS	:	NONE

THIS EXAMINATION PAPER SHOULD NOT BE OPENED UNTIL PERMISSION HAS BEEN GRANTED BY THE INVIGILATOR.

QUESTION 1

The position vector of a moving particle is given by

$$\mathbf{r(s)} = a\cos(\frac{s}{a})\hat{\mathbf{i}} + a\sin(\frac{s}{a})\hat{\mathbf{j}},$$

where s is the arc length parameter and a is a constant. Find

(a) the velocity

(b) the speed

(c) the acceleration

(d) the magnitude of the acceleration

(e) the unit tangent vector

(f) the curvature

(g) the radius of curvature

(h) the unit principal normal

(i) the normal component of acceleration

(j) the unit binormal vector. [20]

QUESTION 2

(a) In polar coordinates (ρ, θ), the position vector of an arbitrary point (x, y) is given by

$$\mathbf{r} = \rho \cos\theta \hat{\mathbf{i}} + \rho \sin\theta \hat{\mathbf{j}}.$$

Show that, in this coordinate system,

(i) the velocity is given by

$$\underline{\mathbf{v}} = \frac{d\mathbf{r}}{dt} = \dot{\rho}\hat{\rho} + \rho\dot{\theta}\hat{\theta}.$$

[5]

(ii) the acceleration is given by

$$\mathbf{a} = \frac{\mathrm{d}\mathbf{v}}{\mathrm{d}t} = (\ddot{\rho} - \rho\dot{\theta}^2)\hat{\rho} + (\rho\ddot{\theta} + 2\dot{\rho}\dot{\theta})\hat{\theta}.$$

[3]

(b) If $\nabla\phi = 2xyz^3\hat{\mathbf{i}} + x^2z^3\hat{\mathbf{j}} + 3x^2yz^2\hat{k}$, find $\phi(x, y, z)$ if $\phi(2, -2, 2) = 6$. [8]

(c) If $\phi = x^2yz^3$ and $\mathbf{A} = xz\mathbf{i} - y^2\mathbf{j} + 2x^2y\mathbf{k}$, find $div(\phi\mathbf{A})$. [4]

QUESTION 3

(a) A train takes time T to perform a journey. It travels for time $\frac{T}{n}$ with uniform acceleration, then for time $(n-2)\frac{T}{n}$ with uniform speed V, and finally for time $\frac{T}{n}$ with constant retardation. Prove that its average speed is

$$(n-1)\frac{V}{n}.$$

If the length of this journey is 64 km, the time taken on the whole journey is 60 minutes, and the uniform speed is 96 km/h, find the time which is occupied in traveling with the uniform speed. [6]

(b) Particle A, initially at rest, is projected from the origin with acceleration $\frac{\sqrt{3}}{2}\hat{\mathbf{i}} + \frac{1}{2}\hat{\mathbf{j}}$. Particle B, at rest at the point $(\sqrt{3}, 0)$, is projected at the same instant with acceleration $\frac{1}{2}\hat{\mathbf{j}}$. Show that the particles collide and that the time of collision is $t = 2$. [7]

(c) A particle moving in a straight line is acted upon by a retarding force of kv^3 per unit mass, where k is a constant and v is the speed. Show that after traveling a distance x, the speed and time taken are given by

$$v = \frac{u}{1 + kux} \quad \text{and} \quad t = \frac{1}{2}kx^2 + \frac{x}{u},$$

where u is the initial speed. [7]

QUESTION 4

(a) A particle of unit mass is thrown vertically upwards with initial speed V. The air resistance at speed v is kv^2 per unit mass, where k is a constant.

(i) Show that H, the maximum height reached, is given by

$$H = \frac{1}{2k} \ln\left(\frac{g + kV^2}{g}\right),$$

and that the time T taken to reach this height is

$$T = \frac{1}{\sqrt{gk}} \tan^{-1}\left[\left(\frac{k}{g}\right)^{\frac{1}{2}} V\right].$$

[7]

(ii) Show that the particle return to the point of projection with speed v^*, where

$$v^* = V\left(\frac{g}{g + kV^2}\right)^{\frac{1}{2}}.$$

[5]

(b) A particle is projected with velocity $\mathbf{u}$ from a point O in a vertical plane through the line of greatest slope of a plane declined at an angle $-\beta$ to the horizontal. After time T, the particle strikes the plane at the point P, at a distance R from O. If $\mathbf{u}$ makes an angle α with the horizontal, and if $|\mathbf{u}| = u$, show that:

(i) $T = \dfrac{2u\sin(\alpha + \beta)}{g\cos\beta}$ and $R = \dfrac{u^2[\sin(2\alpha + \beta) + \sin\beta]}{g\cos^2\beta}$; [5]

(ii) for constant u and β, R is maximum when $\alpha = \dfrac{\pi}{4} - \dfrac{\beta}{2}$. [3]

QUESTION 5

(a) From a point O, at height h above sea level, a particle is projected under gravity with a velocity of magnitude $\frac{3}{2}\sqrt{gh}$. Find the two possible angles of projection if the particle strikes the sea at horizontal distance $3h$ from 0. [8]

(b) Show, by means of the substitution $r = \frac{1}{u}$, that the equation of a particle moving in a force field is

$$\frac{\mathrm{d}^2 u}{\mathrm{d}\theta^2} + u = -\frac{f(1/u)}{mh^2u^2}.$$

[6]

(c) Show that if a central force field is defined by

$$f(r) = -\frac{K}{r^2}, \quad K > 0,$$

that is, an inverse square law of attraction, then the path of the particle is a conic. [6]

QUESTION 6

(a) An inductor of 2 henries, a resistor of 16 ohms, and a capacitor of 0.02 farads are connected in series with a battery of e.m.f. $E = 100\sin(3t)$. At $t \leq 0$ the charge on the capacitor and the current in the circuit are zero. Find the charge and current at any time $t > 0$. [12]

(b) A particle of mass 2 gm moves along the $x-$ axis attracted toward the origin O with a force numerically equal to $8x$. If it is initially at rest at $x = 10$ cm, find its position at any later time t assuming

(i) no other forces act on the particle, [4]

(ii) a damping force numerically equal to 8 times the instantaneous speed acts on the particle. [4]

QUESTION 7

(a) Describe two physical systems that can be modeled by the differential equation

$$\frac{\mathrm{d}^2x}{\mathrm{d}t^2} + 2k\frac{\mathrm{d}x}{\mathrm{d}t} + w_0^2x = f(t),$$

where k and w_0 are fixed constants and $f(t)$ corresponds to an external force. [4]

(b) A 20 kg weight suspended at the end of a vertical spring stretches it 20 cm. Assuming no external forces, find the position of the weight at any time t if initially the weight is

(i) pulled down 10 cm and released, [4]

(ii) pulled down 15 cm and given an initial speed of 105 cm/sec downward.

Find the period and the amplitude in each case. [4]

(c) Solve the mass-spring problem in (b) taking into account an external damping force given in kilograms by $v/7$, where v is the instantaneous speed in cm/sec. [8]

END OF EXAMINATION