UNIVERSITY OF SWAZILAND

SUPPLEMENTARY EXAMINATIONS 2007/2008

B.Sc. / B.Ed. / B.A.S.S. II

TITLE OF PAPER	:	DYNAMICS I
COURSE NUMBER	:	M255
TIME ALLOWED	:	THREE (3) HOURS
INSTRUCTIONS	:	1. THIS PAPER CONSISTS OF SEVEN QUESTIONS. 2. ANSWER ANY FIVE QUESTIONS
SPECIAL REQUIREMENTS	:	NONE

THIS EXAMINATION PAPER SHOULD NOT BE OPENED UNTIL PERMISSION HAS BEEN GRANTED BY THE INVIGILATOR.

QUESTION 1

The position vector of a moving particle is given by

$$\mathbf{r} = 3\cos(3t)\hat{\mathbf{i}} + 3\sin(3t)\hat{\mathbf{j}} + (9t-6)\hat{\mathbf{k}}.$$

Find

(a) the velocity

(b) the speed

(c) the acceleration

(d) the magnitude of the acceleration

(e) the unit tangent vector

(f) the curvature

(g) the radius of curvature

(h) the unit principal normal

(i) the normal component of acceleration

(j) the unit binormal vector. [20]

QUESTION 2

(a) Given the points $A(2,3,1)$, $B(-1,1,2)$ and $C(1,-2,3)$,

(i) show that the acute angle θ which the median to the side AC makes with the side BC is given by

$$\theta = \cos^{-1}(\frac{\sqrt{91}}{14})$$

(ii) find the angle between $\overline{AB}$ and $\overline{BC}$

(iii) find the equation of the plane passing through the three points [10]

(b) If $\phi = x^2yz^3$ and $\mathbf{A} = xz\mathbf{i} - y^2\mathbf{j} + 2x^2y\mathbf{k}$,

find $\nabla \times \mathbf{A}$. [3]

(c) If $\mathbf{A} \cdot (\mathbf{B} \times \mathbf{C}) = (\mathbf{A} \times \mathbf{B}) \cdot \mathbf{C}$ and $(\mathbf{A} \times \mathbf{B}) \times \mathbf{C} = (\mathbf{A} \cdot \mathbf{C})\mathbf{B} - (\mathbf{B} \cdot \mathbf{C})\mathbf{A}$

show that

$$(\mathbf{a} \times \mathbf{b}) \cdot (\mathbf{c} \times \mathbf{d}) = (\mathbf{a} \cdot \mathbf{c})(\mathbf{b} \cdot \mathbf{d}) - (\mathbf{a} \cdot \mathbf{d})(\mathbf{b} \cdot \mathbf{c})$$

[7]

QUESTION 3

(a) A particle starts from rest and moves in a straight line with acceleration $(16-2v^2)$, where v is its speed. Show that the particle has terminal velocity $V = \sqrt{8}$, and find an expression for v in terms of the distance traveled. [5]

(b) A body of unit mass moving in a straight line is projected with speed u from a point at a distance d from the origin. It is acted upon by a force $\frac{k}{x}$, where k is a constant and x is the distance from the origin. Show that

$$x = de^{\frac{u^2-v^2}{2k}},$$

where v is the body's speed. [5]

(c) A particle drops from rest under gravity in a medium which exerts a resistive force of kv per unit mass, where k is a constant and v is the speed. Show that the terminal velocity is given by

$$V = \frac{g}{k}.$$

Also show that the speed v and the distance traveled x at any time t are given by

$$v = V\left(1 - e^{\frac{-gt}{V}}\right)$$

and

$$x = Vt - \left(\frac{V^2}{g}\right)\left(1 - e^{\frac{-gt}{V}}\right).$$

[10]

QUESTION 4

(a) The velocity at time t of a particle moving in a straight line is given by $\frac{(5-t^2)x^2}{10t^2}$, where x is its distance from its starting point. If $x = \frac{5}{3}$ at $t = 1$, show that $x = \frac{10t}{5+t^2}$. [4]

(b) Show that the unit vectors perpendicular to the plane of the vectors $3\hat{\mathbf{i}} - 2\hat{\mathbf{j}} + 4\hat{\mathbf{k}}$ are given by

$$\pm\frac{\left(2\hat{\mathbf{j}} + \hat{\mathbf{k}}\right)}{\sqrt{5}}.$$

[4]

(c) A projectile is launched with initial speed v_0 at an angle α with the horizontal. Find:

(i) The position vector at any time t; [4]

(ii) The time taken by the projectile to reach the highest point; [2]

(iii) The maximum height reached by the projectile; [2]

(iv) The time of flight back to earth; [2]

(v) The range. [2]

QUESTION 5

(a) Prove that the line joining the midpoints of two sides of a triangle is parallel to the third side and has half its length. [8]

(b) In traveling a total distance S, a train accelerates uniformly from rest through a distance pS, then travels with uniform speed V, and finally retards to rest through a distance qS. Show that the average speed $\overline{v}$ for the whole journey is given by

$$\overline{v} = \frac{V}{1+p+q}.$$

[6]

(c) The acceleration of a particle is given by

$$a = \frac{10}{4+5\sqrt{v}},$$

where v meters per second is the speed of the particle at distance x meters from the origin. If the particle started from rest at the origin, find how far it has traveled when it has attained a speed of 25 meters per second. [6]

QUESTION 6

(a) Express $x = -3\cos(2t - \frac{\pi}{2})$ in standard form. [2]

(b) State whether x leads or lags y, and by how much, in the equations $x = -3\cos(2t)$, $y = 4\cos(2t)$. [3]

(c) A 20 kg weight suspended at the end of a vertical spring stretches it 20 cm. Assuming no external forces, find the position of the weight at any time t if initially the weight is

(i) pulled down 10 cm and released,

(ii) pulled down 15 cm and given an initial speed of 105 cm/sec downward.

Find the period and the amplitude in each case. [8]

(d) Solve the mass-spring problem in (c) if an external force given by $F(t) = 20\cos 7t$ is applied for $t > 0$. Give a physical interpretation of what happens as t increases. [7]

<u>QUESTION 7</u>

The following equation could represent the damped vertical motion of a mass supported by a spring and subjected to an external force:

$$\frac{\mathrm{d}^2x}{\mathrm{d}t^2} + \frac{\mathrm{d}x}{\mathrm{d}t} + 36x = 10\cos(wt), \quad \text{for } t > 0,$$

the system being in equilibrium under no force for $t \leq 0$.

(a) Find the period of the free oscillations. [5]

(b) Obtain expressions in terms of w for the amplitude and phase of the forced oscillation. [7]

(c) Find the condition for resonance. [4]

(d) Plot the curve of the amplitude against w for a range $4 \leq w \leq t$. [4]

END OF EXAMINATION