UNIVERSITY OF SWAZILAND

FINAL EXAMINATIONS 2007/8

BSc. / BEd. / B.A.S.S. III

TITLE OF PAPER	:	NUMERICAL ANALYSIS I
COURSE NUMBER	:	M 311
TIME ALLOWED	:	THREE (3) HOURS
INSTRUCTIONS	:	1. THIS PAPER CONSISTS OF SEVEN QUESTIONS. 2. ANSWER ANY FIVE QUESTIONS
SPECIAL REQUIREMENTS	:	NONE

THIS EXAMINATION PAPER SHOULD NOT BE OPENED UNTIL PERMISSION HAS BEEN GRANTED BY THE INVIGILATOR.

QUESTION 1

1. (a) Convert the decimal 3.6 into its binary equivalent. [5 marks]

   (b) Convert the binary $(0.0\overline{0011})_2$ into its decimal equivalent. [5 marks]

   (c) Determine the decimal number that has the following representation in the hypothetical Marc-32 computer.

   0 100 01010 010 0010 0011 1010 0000 0000

   [5 marks]

   (d) Determine the Marc-32 representation of the number

   $$2^{-127} + 2^{-128}$$

   [5 marks]

QUESTION 2

2. (a) Given that

   $$f(x) = x(\sqrt{x+1} - \sqrt{x})$$

   i. find a suitable function $g(x)$ that has been reformulated to be algebraically equivalent to $f(x)$ with the aim of avoiding loss of significance error. [5 marks]

   ii. Compare the results of calculating $f(500)$ and $g(500)$ using six digits and rounding. [5 marks]

   (b) Show that the function

   $$f(x) = e^x - x^2$$

   has exactly one root in the interval $[-1, 0]$ [5 marks]

   (c) Find the number of iterations needed to approximate a solution of the equation $x^3 + x - 4 = 0$ on the interval $[1, 4]$ to an accuracy of 10^{-3} using the bisection method. [5 marks]

QUESTION 3

3. (a) For the scheme $x_{n+1} = x_n + c(x_n^2 - 7)$, find the range of values of c for which convergence to the positive fixed point is guaranteed. For what value of c is convergence quadratic? [10 marks]

(b) The positive root of $f(x) = \alpha - \beta x^2 - x$ with α, $\beta > 0$ is sought and the simple iteration

$$x_{n+1} = \alpha - \beta x_n^2$$

is used. Show that convergence will occur for sufficiently close starting value, provided

$$\alpha\beta < \frac{3}{4}$$

[10 marks]

QUESTION 4

4. (a) Use Lagrange interpolation process to obtain a polynomial of least degree that passes through the points (0,2),(1,2),(2,4) and (3,2). [10 marks]

(b) Given that $f(x) = x^2 + 2x + \sin x$, let our initial guesses for a root be $x_0 = -2.0$ and $x_1 = -2.1$. Apply the secant method twice to obtain a new estimate x_3 for the root of $f(x) = 0$. [5 marks]

(c) Complete the following divided-difference table and use it to obtain a polynomial of degree 3 that interpolate the function values indicated

x	$f[\,]$	$f[\,,\,]$	$f[\,,\,,\,]$	$f[\,,\,,\,,\,]$
-1	2			
1	-4		2	
3	6			
		2		
5	10			

[5 marks]

QUESTION 5

5. (a) i. Determine A, B and C such that the quadrature formula

$$\int_0^1 f(x)\,dx = Af\left(\frac{1}{4}\right) + Bf\left(\frac{3}{4}\right) + Cf(1)$$

is exact for the polynomials of as high a degree as possible. [6 marks]

ii. Use the Gaussian Quadrature rule in (i) to approximate the integral

$$\int_0^1 x^4\,dx$$

and compare your result against the exact value of the integral. [4 marks]

(b) Solve the system of equations

$$\begin{aligned} 8x_2 + 2x_3 &= -7 \\ 3x_1 + 5x_2 + 2x_3 &= 8 \\ 6x_1 + 2x_2 + 8x_3 &= 26 \end{aligned}$$

using the **LU** decomposition method. [10 marks]

QUESTION 6

6. (a) Evaluate the integral $\int_0^1 e^{-x^2}\,dx$ using the Trapezoidal rule with $h = 0.25$. [4 marks]

(b) Evaluate $\int_2^6 \frac{x}{1+x}\,dx$ using the Simpson rule with $h = 1$ and calculate the error against the exact value of the integral to four decimal places. [6 marks]

(c) Find the constants c_0, c_1 and x_1 so that the quadrature formula

$$\int_0^1 f(x)\,dx = c_0 f(0) + c_1 f(x_1)$$

is exact for polynomials of as high a degree as possible. [10 marks]

QUESTION 7

7. (a) Let $p_2(x)$ be the quadratic polynomial interpolating $f(x)$ at $(0, f(0))$, $(h, f(2h))$ and $(2h, f(2h))$.

   i. Write down the Lagrange representation of $p_2(x)$. [4 marks]

   ii. By integrating $p_2(x)$ between 0 and $3h$, derive the following numerical integration rule that approximates

$$I = \int_0^{3h} f(x)\, dx$$

   that is, show that

$$I \approx \frac{3h}{4}[f(0) + 3f(2h)].$$

[6 marks]

(b) Given the points (-3,-1), (0,2) and (3,-2), construct a forward difference table; hence deduce the polynomial of degree ≤ 2 that goes through the points in Newton form. [10 marks]