UNIVERSITY OF SWAZILAND

SUPPLEMENTARY EXAMINATIONS 2008

BSc. / BEd. / B.A.S.S. III

TITLE OF PAPER	:	NUMERICAL ANALYSIS I
COURSE NUMBER	:	M 311
TIME ALLOWED	:	THREE (3) HOURS
INSTRUCTIONS	:	1. THIS PAPER CONSISTS OF SEVEN QUESTIONS. 2. ANSWER ANY FIVE QUESTIONS
SPECIAL REQUIREMENTS	:	NONE

THIS EXAMINATION PAPER SHOULD NOT BE OPENED UNTIL PERMISSION HAS BEEN GRANTED BY THE INVIGILATOR.

QUESTION 1

1. (a) Convert the following decimal numbers to their binary equivalent

   i. 0.6 [5 Marks]

   ii. $\frac{17}{16}$ [5 Marks]

   (b) Convert the following binary numbers to their decimal equivalent

   i. $(11011.01)_2$ [5 Marks]

   ii. $(0.0\overline{0011})_2$ [5 Marks]

QUESTION 2

2. (a) Given that

$$f(x) = \frac{e^x - 1 - x}{x^2}$$

find a suitable function $g(x)$ that has been reformulated to be algebraically equivalent to $f(x)$ with the aim of avoiding loss of significance error. [5 marks]

(b) Compare the results of calculating $f(0.01)$ and $g(0.01)$ using six digits and rounding. [5 marks]

(c) For the scheme $x_{n+1} = x_n + c(x_n^2 - 7)$, find the range of values of c for which convergence to the positive fixed point is guaranteed. For what value of c is convergence quadratic? [10 marks]

QUESTION 3

3. (a) Find the **LU** factorization of matrix A

$$A = \begin{pmatrix} 1 & 1 & -2 \\ 1 & 3 & -1 \\ 1 & 5 & 1 \end{pmatrix}$$

[10 marks]

(b) Show that the Newton's iteration for finding the reciprocal of a real number c, is $x_{n+1} = x_n(2 - cx_n)$ and show that the iteration converges for $x_0 \in [1/2c, 3/2c]$ [10 marks]

QUESTION 4

4. (a) Use the two-point Gaussian Quadrature rule,

$$\int_{-1}^{1} f(x)\,dx \approx f\left(\frac{-\sqrt{3}}{3}\right) + f\left(\frac{\sqrt{3}}{3}\right),$$

to approximate the integral

$$\int_{0}^{1} x^2 e^{-x}\,dx$$

and compare your result against the exact value of the integral. [10 marks]

(b) Evaluate the integral $\int_{0}^{1} xe^{-x}\,dx$ analytically correct to four decimal places. Use the trapezoidal rule with $h = 0.2$ and the Simpson's rule with $h = 0.25$ to compute the same integral. Compare the errors. [10 marks]

QUESTION 5

5. Given the points (-2,-1),(-1,3),(0,1),(1,-1) and (2,3)

(a) Construct a forward difference table. [10 marks]

(b) Prove that the polynomial of degree ≤ 4 that goes through the points in Newton form is

$$x^3 - 3x + 1$$

[10 marks]

QUESTION 6

6. Consider the function $f(x) = x^3 + 4x^2 - 10$.

(a) Show that $f(x)$ has exactly one root in [1, 2]. [6 marks]

(b) By performing 4 iterations of the bisection method, show that this root lies in the interval [1.3125, 1.375]. [8 marks]

(c) How many iterations would be required to locate this zero to a tolerance of 10^{-5}? [6 marks]

QUESTION 7

7. (a) Show that the iteration scheme

$$\alpha_{n+1} = \frac{\alpha_n^2 - a\alpha_n + a^2 + 5a}{\alpha_n + 5}$$

converges to the fixed point a quadratically (i.e order of convergence is 2) for all $a \neq -5$. [10 marks]

(b) A certain 3×3 matrix A has an **LU** decomposition with

$$L = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ \frac{2}{3} & -\frac{1}{3} & 1 \end{pmatrix} \qquad U = \begin{pmatrix} 3 & 2 & 5 \\ 0 & -2 & 1 \\ 0 & 0 & -5 \end{pmatrix}$$

Solve the system $Ax = b$ where $b = [5, 15, 10]^T$. [10 marks]