UNIVERSITY OF SWAZILAND

SUPPLEMENTARY EXAMINATIONS 2008

BSc. / BEd. / B.A.S.S. IV

TITLE OF PAPER	:	NUMERICAL ANALYSIS II
COURSE NUMBER	:	M 411
TIME ALLOWED	:	THREE (3) HOURS
INSTRUCTIONS	:	1. THIS PAPER CONSISTS OF SEVEN QUESTIONS. 2. ANSWER ANY FIVE QUESTIONS
SPECIAL REQUIREMENTS	:	NONE

THIS EXAMINATION PAPER SHOULD NOT BE OPENED UNTIL PERMISSION HAS BEEN GRANTED BY THE INVIGILATOR.

QUESTION 1

1. Use the modified Euler's method with $h = 0.25$ to solve

$$y'' + 2y' + y = e^{-x}$$

subject to $y(0) = 1$ and $y'(0) = 2$ and approximate $y(0.25)$ and $y'(0.25)$. [20 marks]

QUESTION 2

2. (a) Solve the following initial value problem using the 4th order Runge-Kutta method with $h = 0.1$ to estimate $y(0.1)$ and compare the result with the exact solution.

$$\frac{dy}{dx} = -2xy \qquad \text{with} \qquad y(0) = 1$$

[10 marks]

(b) Use Newton's interpolating formula

$$f(x, y) \approx f_{k-1} + \frac{(x - x_{k-1})}{h} \Delta f_{k-1}$$

to derive the Adams 2-step formula

$$y_{k+1} = y_k + h\left(\frac{3}{2} f_k - \frac{1}{2} f_{k-1}\right)$$

for integrating over the interval $[k,\, k+1]$, assuming that information at the preceding points x_{k-1} and x_k is known.

[10 marks]

QUESTION 3

3. (a) Discuss the consistency and zero-stability of the following scheme

$$y_{n+1} = 5y_{n-1} - 4y_n + h[4f_n + 2f_{n-1}]$$

[10 marks]

(b) Use Taylor series method to solve the following system of ordinary differential equations.

$$\begin{aligned} \frac{dx}{dt} &= xy + t \,, \qquad x(0) = 1 \\ \frac{dy}{dt} &= ty + x \,, \qquad y(0) = -1 \end{aligned}$$

[10 marks]

QUESTION 4

4. Find the second degree least-squares polynomial approximation of the form $P_2(x) = a_o + a_1 x + a_2 x^2$. that best fits the through the following experimental data.

x	1	2	3	4	5	6
y	1	3	4	3	4	2

[20 marks]

QUESTION 5

5. (a) Consider the second order initial value problem

$$\frac{d^2x}{dt^2} + 4\frac{dx}{dt} + 5x = 0 \quad \text{with} \quad x(0) = 3 \quad \text{and} \quad x'(0) = 5$$

write down the equivalent system of two first order equations. [6 marks]

(b) Use the Taylor method of order 4 to solve

$$\frac{dy}{dt} = \frac{(t-y)}{2}$$

on [0,3] with $y(0) = 1$ and $h = 0.25$ and compare the approximate solution with the exact solution. [7 marks]

(c) Use the 4th order Runge-Kutta method with $h = 0.25$ to solve

$$\frac{dy}{dt} = \frac{(t-y)}{2}$$

on [0,3] with $y(0) = 1$ and compare the approximate solution with the exact solution. [7 marks]

QUESTION 6

6. Determine the system of four (4) equations in four unknowns for computing approximations for the Laplace equation

$$u_{xx} + u_{yy} = 0$$

with $h = 1,\ k = 1$ in the rectangle $R = \{(x, y) :\ 0 \le x < 3,\ 0 \le y \le 3\}$. The boundary values are

$$u(x,0) = 10; \qquad \text{and} \qquad u(x,3) = 90 \qquad 0 \le x \le 3;$$

$$u(0,y) = 70; \qquad \text{and} \qquad u(3,y) = 0; \qquad 0 \le y \le 3$$

[20 marks]

QUESTION 7

7. Consider the parabolic differential equation

$$\frac{\partial u}{\partial t} - \alpha^2 \frac{\partial^2 u}{\partial x^2} = 0, \quad 0 \le x \le 1, \quad t > 0$$

$$u(0,t) = 0, \quad u(1,t) = 0, \quad t > 0$$

$$u(x,0) = \cos 2\pi x, \quad 0 \le x \le 1$$

If an $O(k^2 + h^2)$ numerical method is constructed using the central difference quotient to approximate u_t and the usual difference quotient to approximate u_{xx},

(a) Write down the finite difference scheme for the problem. [10 marks]

(b) Show that the method has the matrix form

$$\mathbf{u}^{(j+1)} = \mathbf{u}^{(j-1)} + A\mathbf{u}^{(j)} \quad \text{for each} \quad j = 0, 1, 2, \ldots$$

where $\mathbf{u}^{(j)} = (u_{1,j}, u_{2,j}, \ldots, u_{m-1,j})^T$ and A is a tri-diagonal matrix. [10 marks]