UNIVERSITY OF SWAZILAND

FINAL EXAMINATIONS 2008

BSc. / BEd. / B.A.S.S. IV

TITLE OF PAPER	:	FLUID DYNAMICS
COURSE NUMBER	:	M 455
TIME ALLOWED	:	THREE (3) HOURS
INSTRUCTIONS	:	1. THIS PAPER CONSISTS OF SEVEN QUESTIONS. 2. ANSWER ANY FIVE QUESTIONS
SPECIAL REQUIREMENTS	:	NONE

THIS EXAMINATION PAPER SHOULD NOT BE OPENED UNTIL PERMISSION HAS BEEN GRANTED BY THE INVIGILATOR.

QUESTION 1

1. (a) If $\mathbf{q} = (2x + t)\mathbf{i} + (y - 2t)\mathbf{j}$, find the streamline through the point (1,1) at $t = 1$. [4 marks]

   (b) i. Prove that the following velocity field represent possible motion of an incompressible fluid and find the equations of the streamlines.

$$\mathbf{q} = \left[\left(\frac{a^2}{r^2} - 1 \right) \cos\theta, \left(1 + \frac{a^2}{r^2} \right) \sin\theta, 0 \right] \qquad \text{(cylindrical)}$$

[8 marks]

   ii. Does the velocity field in (i) represent an irrotational fluid? If so, find the velocity potential. [8 marks]

QUESTION 2

2. (a) Find the acceleration of the fluid whose velocity field is given by

$$\mathbf{q} = 4tx\mathbf{i} - 2t^2y\mathbf{j} + 4xz\mathbf{k}$$

[6 marks]

   (b) The velocity potential for a two dimensional flow is given by

$$\phi = -k \tan^{-1}\left(\frac{y}{x}\right)$$

   where k is a positive constant.

   Find the velocity field for the flow. [7 marks]

   (c) Verify that the velocity field $\mathbf{q} = x^2\mathbf{i} - 2xy\mathbf{j}$ could describe the flow of an incompressible fluid. If the fluid is inviscid and the body force per unit mass is $\mathbf{F} = -xy^2\mathbf{i} + x^2y\mathbf{j}$, show that the pressure distribution is given by [7 marks]

$$p = p_0 - \frac{\rho}{2}x^2(x^2 + y^2)$$

QUESTION 3

3. (a) Find the velocity components corresponding to the complex potential,

$$w(z) = U\left(z + \frac{a^2}{z}\right) + \frac{ik}{2\pi}\ln z$$

[6 marks]

(b) Find the complex potential $w(z)$ corresponding to the velocity potential

$$\phi = (x+1)^2 - y^2$$

[6 marks]

(c) The velocity potential for a steady incompressible, irrotational flow with circulation around a fixed cylinder of radius a is given in cylindrical polar coordinates by

$$\phi = Ur\left(1 + \frac{a^2}{r^2}\right)\cos\theta + \frac{k\theta}{2\pi}$$

where U is the uniform speed at infinity. Find the corresponding stream function ψ. [8 marks]

QUESTION 4

4. (a) Fluid flows out of a circular tank of radius A though a small circular hole of radius a located in the bottom of the tank. Assuming that the flow is steady and that the pressure both at the free surface and the exit hole is atmospheric, show that the time T required to empty the tank is given by

$$T = \left[\frac{A^4}{a^4} - 1\right]^{\frac{1}{2}}\left(\frac{2h_0}{g}\right)^{\frac{1}{2}}$$

where the height $h = h_0$ when $t = 0$ and g is acceleration due to gravity. [10 marks]

(b) Two doublets, each of strength μ, are situated at the points $z = \pm a$, and two other doublets, each of strength 2μ are situated at the points $z = \pm ia$. Each of the four doublets has its axis tangential to the circle $|z| = a$ and pointing in the positive sense of rotation. Show that $w(z)$, the total complex velocity potential of the four doublets is given by

$$w(z) = \frac{2\mu ia(3a^2 - z^2)}{z^4 - a^4}$$

[10 marks]

QUESTION 5

5. (a) Consider the steady axisymmetric flow in a pipe of constant cross-section. Assume that the flow is one-dimensional along the axis of the pipe of radius R and that there is no velocity swirl. Given that $\mathbf{q} = (0, 0, q_z(r))$ prove that the velocity profile for the flow is given by

$$q_z = \frac{1}{4\mu}\frac{dp}{dz}(r^2 - R^2)$$

[10 marks]

(b) Consider the motion of a fluid between two parallel plates. If the lower surface is held fixed and coincides with the surface $y = 0$ while the upper surface, located at $y = 1$ is moved with a constant speed α in a plane parallel to the x-axis. Assuming that the motion of the fluid is predominantly parallel to the x-axis, i.e $\mathbf{q} = (f(y), 0, 0)$. Determine the velocity profile $f(y)$ of the fluid motion. [10 marks]

QUESTION 6

6. Consider the boundary layer equations in the form [20 marks]

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (1)$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu\frac{\partial^2 u}{\partial y^2} \quad (2)$$

with boundary conditions

$$u = 0 \;\;,\;\; v = 0 \;\;\text{on y} = 0 \;\;\text{and}\;\; u = U \;\;\text{on}\;\; y = \infty$$

Use the similarity transformation

$\eta = y\sqrt{\frac{U}{\nu x}}$ and the stream function formulation $\psi = \sqrt{\nu x U} f(\eta)$ (where ν is the dynamic viscosity) to show that equation (2) and the boundary conditions can be transformed into

$$2\frac{d^3 f}{d\eta^3} + f\frac{d^2 f}{d\eta^2} = 0$$

with boundary conditions

$$f = 0 \;\;,\;\; \frac{df}{d\eta} = 0 \;\;\text{on}\; \eta = 0 \;\;\text{and}\;\; \frac{df}{d\eta} = -1 \;\;\text{on}\; \eta = \infty$$

QUESTION 7

7. In the $z = x + iy$ plane, a line vortex of strength $m > 0$, is placed at $z = c$ and another, of strength $-m$, at $z = -c$, where c is a real positive number. Both vortices are held fixed at these locations. Write down the complex potential w for this flow and show that the stream function ψ and the [20 marks] velocity potential ϕ are given respectively by

$$\psi = \frac{m}{4\pi}\log\frac{(x-c)^2 + y^2}{(x+c)^2 + y^2} \quad \text{and}$$

$$\phi = -\frac{m}{2\pi}\tan^{-1}\frac{2cy}{x^2 + y^2 - c^2}$$

hint: You may set $z - c = r_1 e^{i\theta_1}$ and $z + c = r_2 e^{i\theta_2}$, and use $\tan^{-1} A + \tan^{-1} B = \tan^{-1}\left(\frac{A+B}{1-AB}\right)$