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# University of Swaziland



Final Examination, December 2008

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BSc I, EEng I, BEd I

**Title of Paper** : Algebra, Trig. and Analytic Geometry

**Course Number** : M111

**Time Allowed** : Three (3) hours

**Instructions** :

1. This paper consists of SEVEN questions.
2. Each question is worth 20%.
3. Answer ANY FIVE questions.
4. Show all your working.

THIS PAPER SHOULD NOT BE OPENED UNTIL PERMISSION HAS BEEN GIVEN BY THE INVIGILATOR.

### Question 1

(a) Divide

$$\frac{x^5 + 12x^3 - 21x - 20}{x^2 - 2x + 2}. \quad [8 \text{ marks}]$$

(b) Use the binomial theorem to find the exact value of  $(2 - \sqrt{2})^5$ . [7 marks]

(c) Evaluate

$$\begin{vmatrix} 2 & 0 & -3 & 0 \\ -1 & 2 & 0 & 1 \\ 4 & 0 & 0 & 3 \\ 0 & 4 & 2 & 0 \end{vmatrix}. \quad [5 \text{ marks}]$$

### Question 2

(a) Given that  $a, b, c$  are in arithmetic progression, prove that  $\ln a, \ln b, \ln c$  are in geometric progression. [5 marks]

(b) Find the exact value of the infinite sum

$$2 - \sqrt{2} + 1 - \dots. \quad [5 \text{ marks}]$$

(c) Find the fourth roots of

$$-81. \quad [10 \text{ marks}]$$

### Question 3

(a) Solve

i.  $\log_2(x + 2) = 3 - \log_2(x - 5)$  [4 marks]

ii.  $4^x - 3 \cdot 2^{x+1} + 8 = 0.$  [6 marks]

(b) Find the first five terms of the binomial expansion of

$$(1+x)\sqrt{1-2x}. \quad [10 \text{ marks}]$$

#### Question 4

(a) Prove by mathematical induction

$$1^3 + 3^3 + 5^3 + \dots + (2n-1)^3 = n^2(2n^2-1), \quad n = 1, 2, \dots. \quad [10 \text{ marks}]$$

(b) Find all values of  $x$  in the interval  $0 \leq x < 2\pi$ , satisfying

$$\cos 2x + \cos x + \cos^2 x = -\sin^2 x. \quad [6 \text{ marks}]$$

(c) Find the centre and radius of the circle with equation

$$5x^2 + 5y^2 + 20x + 50y + 129 = 0. \quad [4 \text{ marks}]$$

#### Question 5

(a) Find the term independent of  $x$  in the expansion of

$$\left(x - \frac{1}{\sqrt{x}}\right)^{12}. \quad [4 \text{ marks}]$$

(b) Prove

$$\sin^6 \varphi + \cos^6 \varphi = 1 - 3 \sin^2 \varphi \cos^2 \varphi. \quad [10 \text{ marks}]$$

(c) Evaluate

$$\frac{5+5i}{3-4i} + \frac{20}{4+3i}$$

and express your answer in the form  $a + ib$ . [6 marks]

### Question 6

- (a) Use the rational root theorem and synthetic division to find all the roots of

$$x^4 + x^3 - 19x^2 + 11x + 30 = 0. \quad [10 \text{ marks}]$$

- (b) Solve

$$z^4 + z^2 + 1 = 0. \quad [10 \text{ marks}]$$

### Question 7

- (a) Use Cramer's rule to solve the system

$$4x + 4y - 3z = 3$$

$$2x + 3y + 2z = -4$$

$$3x - y + 4z = 4.$$

[13 marks]

- (b) Evaluate

$$\left(\sqrt{3} - i\right)^{10},$$

and express in the form  $a + ib$ .

[7 marks]

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