UNIVERSITY OF SWAZILAND

FINAL EXAMINATIONS 2008/2009

BSc. / BEd. / B.A.S.S. I

TITLE OF PAPER	:	INTRODUCTION TO CALCULUS
COURSE NUMBER	:	M 115
TIME ALLOWED	:	THREE (3) HOURS
INSTRUCTIONS	:	1. THIS PAPER CONSISTS OF SEVEN QUESTIONS.
		2. ANSWER ANY FIVE QUESTIONS
		3. ONLY NON-PROGRAMMABLE CALCULATORS MAY BE USED.
SPECIAL REQUIREMENTS	:	NONE

THIS EXAMINATION PAPER SHOULD NOT BE OPENED UNTIL PERMISSION HAS BEEN GRANTED BY THE INVIGILATOR.

QUESTION 1

(a) Find the limit

$$\lim_{x \to \infty} \frac{1 - 2x - 2x^2}{2x^3 - 3x + 1}$$

[4]

(b) Find the equation for the tangent to the curve at the given point:

$$f(x) = x^3 - 3x \quad \text{at} \quad p(2, 2)$$

(c) Find the derivative of y with respect to x for $y = x^x$. [4]

(d) Find the first and second derivative of the following

$$f(x) = x^2 \cos x$$

[8]

QUESTION 2

(a) Find $\frac{dy}{dx}$ for the following, in as simplified a form as possible

(i) $y = (x^2 + 1)^{\frac{3}{2}} + \sqrt{x^2 + 1}$ [5]

(ii) $y^3 + y + x^2 + x = 0$ [5]

(b) Evaluate the following integrals

(i) $\int \dfrac{x}{(x-1)^2(x+1)}$ [5]   (ii) $\int x(\ln x)^2 dx$ [5]

QUESTION 3

(a) Evaluate the following integral

$$\int \frac{dx}{(4 + x^2)^2}$$

[6]

(b) Find the area bounded by the curve $y = 4x - x^2$ and the straight line $y = 3x$.   [7]

(c) Use partial fractions to evaulate

$$\int \frac{x^2 + x + 1}{x^3 - x^2 - x + 1} dx$$

[7]

3

QUESTION 4

(a) Evaluate

(i) $\int \sin^4 x \cos^3 x \, dx$ [5]   (ii) $\int \dfrac{x^2}{(9 - x^2)^{\frac{3}{2}}} dx$ [5]

(b) Find $\dfrac{dy}{dx}$ for $y = x^2 \arcsin\left(\dfrac{1}{x}\right)$ [5]

(c) Use Leibnitz's rule to find $\dfrac{d^4 y}{dx^4}$ for $y = x^5 \ln x$ [5]

QUESTION 5

(a) Derive a reduction formula for $\int \cos^n x \, dx$ [5]

and hence evaluate $\int \cos^3 x \, dx$ [5]

(b) Use logarithmic differentiation to find $\dfrac{dy}{dx}$

(i) $y = x^x$ [5]   (ii) $y = x^{\sin x}$ [5]

4

QUESTION 6

(a) Use the definition (not formulas) to compute the derivative for the following

(i) $f(x) = \frac{x-1}{x+2}$ [5]   (ii) $y = \sqrt{x+1}$ [5]

(b) Evaluate the definite integral

$$\int_3^4 x(x^2 + 3)^8 dx$$

[5]

(c) Use integration by parts to evaluate $\int x^2 \cos x \, dx$ [5]

QUESTION 7

(a) Use implicit differentiation to find $\frac{dy}{dx}$ given that

$$\cos(xy) + x^2 y + y = 0$$

[5]

(b) Calculate the area bounded by the curves $y = x^2 - 1$ and $y = 1 - x^2$ [10]

(c) Find the integral $\int \sin 5x \cos 2x \, dx$ [5]