# UNIVERSITY OF SWAZILAND

## SUPPLEMENTARY EXAMINATIONS 2008/2009

## BSc. / BEd. / B.A.S.S. I

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| TITLE OF PAPER | : | INTRODUCTION TO CALCULUS |
| COURSE NUMBER | : | M 115 |
| TIME ALLOWED | : | THREE (3) HOURS |
| INSTRUCTIONS | : | 1. THIS PAPER CONSISTS OF SEVEN QUESTIONS.<br>2. ANSWER ANY FIVE QUESTIONS<br>3. ONLY NON-PROGRAMMABLE CALCULATORS MAY BE USED. |
| SPECIAL REQUIREMENTS | : | NONE |

THIS EXAMINATION PAPER SHOULD NOT BE OPENED UNTIL PERMISSION HAS BEEN GRANTED BY THE INVIGILATOR.

## QUESTION 1

(a) Evaluate the following integrals using trig substitution

(i) $\int \frac{1}{\sqrt{4-9x^2}}dx$ [5] (ii) $\int \frac{2x-3}{x^2+6x+13}dx$ [5]

(b) Evaluate the following definite integrals

(i) $\int_{\frac{1}{2}}^{\frac{e}{2}} \frac{ln(2x)}{x}dx$ [5] (ii) $\int_2^4 \frac{x}{2+5x^2}dx$ [5]

## QUESTION 2

(a) Evaluate the integral $\int \frac{\cos x}{1-\sin x}dx$ [5]

(b) Evaluate $\lim_{x\to 1} \frac{2x^2-3x+1}{x-1}$ [5]

(c) Use the definition to evaluate the derivative of

(i) $f(x) = \sqrt{x}$ [5] (ii) $f(x) = \frac{1}{1-x^2}$ [5]

## QUESTION 3

(a) Use implicit differentiation to find $\frac{dy}{dx}$ given that $e^x - e^y = e^{x-y}$ [5]

(b) Use the chain rule to find $\frac{dy}{dx}$ given that $y = 2u^2$ and $u = x^2 - 1$ [5]

(c) Evaluate the following integrals

(i) $\int \frac{x}{\sqrt{1-x}} dx$ [5] (ii) $\int x \ln(3x) dx$ [5]

## QUESTION 4

(a) Derive a reduction formula for $\int \sin^n x dx$ [5]

(b) Use (a) to evaluate

$$\int \sin^5 x dx$$

[5]

(c) Find $\frac{dy}{dx}$ for $x \cos y = \sin(x + y)$ [5]

(d) Find $\frac{dy}{dx}$ for $y = x^2 arcsec(\frac{2}{x})$ [5]

## QUESTION 5

(a) Evaluate the following integrals

(i) $\int \frac{\cos^3 3x}{\sin^2 3x} dx$ [5] (ii) $\int \sin^4 x \cos^2 x dx$ [5]

(b) Find the derivative $\frac{dy}{dx}$ for each of the following

(i) $y = 2cosec(3x^2) + 5x \sin x$ [5]

(ii) $y = \cos 2x \sinh 2x$ [5]

## QUESTION 6

(a) (i) Find the expression for $\frac{d^n y}{dx^n}$ (n is a positive integer)for

$$y = \frac{1}{1-2x}$$

[5]

(b) Integrate

(i) $\int \frac{d\theta}{1+\cos\theta}$ [5] (ii) $\int \frac{x^3}{x^2-2x+1} dx$ [5]

(c) Find $\frac{d^2y}{dx^2}$ for $y = e^x \cos 4x$ [5]

## QUESTION 7

(a) Evaluate the following limits

(i) $\lim_{x \to 2} \dfrac{4 - x^2}{3 - \sqrt{x^2 + 5}}$ [5] (ii) $\lim_{x \to 1} \dfrac{x^3 - 1}{4x^3 - x - 3}$ [5]

(b) Show that each of the specified function satisfies the given partial differential equations

(i) $z = \sqrt{x^2 + y^2}$ satifies $x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} = z$ [5]

(ii) $f = e^{\frac{x}{y}} \sin(\frac{x}{y}) + e^{\frac{y}{x}} \cos(\frac{y}{x})$ satifies $xf_x + yf_y = 0$ [5]