UNIVERSITY OF SWAZILAND

FINAL EXAMINATIONS 2008/2009

BSc. / BEd. / B.A.S.S. II

TITLE OF PAPER	:	CALCULUS 1
COURSE NUMBER	:	M 211
TIME ALLOWED	:	THREE (3) HOURS
INSTRUCTIONS	:	1. THIS PAPER CONSISTS OF SEVEN QUESTIONS. 2. ANSWER ANY FIVE QUESTIONS 3. ONLY NON-PROGRAMMABLE CALCULATORS MAY BE USED.
SPECIAL REQUIREMENTS	:	NONE

THIS EXAMINATION PAPER SHOULD NOT BE OPENED UNTIL PERMISSION HAS BEEN GRANTED BY THE INVIGILATOR.

QUESTION 1

(a) For the function $f(x) = \frac{x^2-3x}{(x+3)^2}$, show that

(i) $f'(x) = \frac{9x-9}{(x+3)^3}$, and

(ii) $f''(x) = \frac{-18x+54}{(x+3)^4}$

Hence find the

(iii) intervals of increase and decrease,

(iv) stationary points,

(v) intervals of upward and downward concavity,

(vi) inflection points,

(vii) intercepts.

Using the information above, sketch the graph of $f(x)$. [20 marks]

QUESTION 2

(a) (i) State the First Derivative Theorem for local extreme Values.

(ii) Define a critical point c of a function $f(x)$, hence find the absolute maximum and minimum values of

$f(x) = 8x - x^4$ on $[-2, 1]$. [10 marks]

(b) (i) State (without proof) the Mean Value Theorem.

(ii) Show that if $f'(x) = 0$ for all $x \epsilon (a, b)$, then $f(x)$ is a constant function over (a, b). [10 marks]

QUESTION 3

(a) (i) State the Alternating Series Test Theorem.

(ii) Use this test to investigate the series $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{2n^3 - 1}$. [10 marks]

(b) (i) State the Limit Comparison Test Theorem.

(ii) Use this test to investigate the following series:

1. $\sum_{n=1}^{\infty} \frac{n}{n^2 + 1}$ 2. $\sum_{n=1}^{\infty} \frac{1}{2^n - 1}$. [10 marks]

QUESTION 4

Use L'Hopital's Rule to evaluate the limit of the following functions:

(a) $\lim_{x \to \infty} \frac{5x^2 - 3x}{7x^2 + 1}$ (b) $\lim_{t \to 0} \frac{\sin t^2}{t}$

(c) $\lim_{x \to 0} \left(\frac{1}{\sin x} - \frac{1}{x} \right)$ (d) $\lim_{x \to 0^+} (1 + x)^{\frac{1}{x}}$ [20 marks]

QUESTION 5

(a) (i) State the Sandwich Theorem for Sequences.

(ii) Use the theorem above to find the limit of the sequence

$a_n = \frac{\cos n}{n^2}$. [6 marks]

(b) Find the area of the surface generated by revolving

the curve $y = 2\sqrt{x}$, $1 \leq x \leq 2$, about the x-axis. [5 marks]

(c) Derive the formula of the length of a plane curve $y = f(x)$

and hence find the length of the curve, $y = \frac{1}{3}(x^2 + 2)^{\frac{3}{2}}$ from

$x = 0$ to $x = 3$. [9 marks]

QUESTION 6

(a) Define the **radius** and **interval of convergence** of a power series of the form $\sum_{n=0}^{\infty} c_n(x-a)^n$, hence find the radius and interval of convergence of the power series: $\sum \frac{(-1)^n n! x^n}{n^n}$ [8 marks]

(b) Write down the first four terms in the Binomial series

for $\sqrt{9-x}$. [5 marks]

(c) Find the Taylor Series for $f(x) = \ln(x)$ about $x = 2$. [7 marks]

QUESTION 7

(a) Prove that the volume of a sphere generated by rotating the semi-circle $x^2 + y^2 \leq a^2 \quad (y \geq 0)$ about the x-axis is given by $\frac{4}{3}\pi a^3$. [6]

(b) Find the volume of the solid of revolution obtained by rotating the area bounded by $y = 1 - x^2$, and $y = 4 - 4x^2$ about

(i) $x-$axis (ii) the line $y = -1$. [14 marks]

END OF EXAMINATION