UNIVERSITY OF SWAZILAND

SUPPLEMENTARY EXAMINATIONS 2008/9

BSc. / BEd. / B.A.S.S. II

TITLE OF PAPER	:	CALCULUS 1
COURSE NUMBER	:	M 211
TIME ALLOWED	:	THREE (3) HOURS
INSTRUCTIONS	:	1. THIS PAPER CONSISTS OF SEVEN QUESTIONS. 2. ANSWER ANY FIVE QUESTIONS 3. ONLY NON-PROGRAMMABLE CALCULATORS MAY BE USED.
SPECIAL REQUIREMENTS	:	NONE

THIS EXAMINATION PAPER SHOULD NOT BE OPENED UNTIL PERMISSION HAS BEEN GRANTED BY THE INVIGILATOR.

QUESTION 1

(a) Let $f(x) = e^x(x^2 - 2x)$. Show that the hypotheses of Rolle's Theorem are satisfied in the interval $[0, 2]$. Find the number c whose existence is guaranteed by the Theorem. [8 marks]

(b) Find the extreme values of the function $f(x) = x^4 - 2x^2 + 5$ on the interval $[-2, \frac{1}{2}]$. [6 marks]

(c) Find the sum of the series: $\sum_{n=1}^{\infty} \frac{2}{n(n+2)}$. [6 marks]

QUESTION 2

(a) (i) **Define** concavity and **state** the test for concavity.

(ii) Determine the open intervals on which the graph of $f(x) = 6(x^2 + 3)^{-1}$ is concave upward or downward. [10 marks]

(b) **State** the Second Derivative Test theorem and **apply** this theorem to find the relative extrema of the function $f(x) = x^3 - 3x^2 + 3$. [10 marks]

QUESTION 3

(a) Let a_n be a sequence in which $a_n = \frac{n}{2^{n+2}}$ for $n \geq 1$.

(i) Determine if the sequence is monotone and whether it is bounded.

(ii) State whether the sequence is convergent from your answers to (i) above. [10 marks]

(b) Evaluate the limit:

(i) $\lim_{x \to \frac{\pi}{2}} \frac{1 - \sin x}{1 + \cos 2x}$, (ii) $\lim_{x \to 0^+} (1+x)^{\frac{1}{x}}$. [10 marks]

QUESTION 4

(a) Compute the sum of the series $\sum_{n=1}^{\infty} \frac{2n+1}{n^2(n+1)^2}$ if it converges. [6 marks]

(b) Determine whether the given series converges or diverges.

State clearly the test you apply in each case.

(i) $\sum_{n=1}^{\infty} \left(\frac{n+1}{n}\right)^n$ (ii) $\sum_{n=1}^{\infty} \frac{100.3^{n+1}}{n!}$ (ii) $\sum_{n=1}^{\infty} \frac{5}{5n-1}$. [14 marks]

QUESTION 5

(a) Use Maclaurin's series for $\sin x$ to evaluate
$\lim_{x\to 0} \dfrac{x+\sin x}{x(x+1)}$. [6 marks]

(b) Use a Binomial series to evaluate $\sqrt{1.01}$ correct to six decimal places. [5 marks]

(c) Find the radius and interval of convergence of the following power series:

(i) $\sum_{n=1}^{\infty} \dfrac{(-1)^n n}{4^n}(x+3)^n$ (ii) $\sum_{n=0}^{\infty} \dfrac{nx^n}{2^{n+1}}$. [9 marks]

QUESTION 6

(a) Prove that the volume of a sphere generated by rotating the semi-circle $x^2+y^2 \leq a^2$ $(y \geq 0)$ about the x-axis is given by $\frac{4}{3}\pi a^3$. [6]

(b) Find the volume of the solid of revolution obtained by rotating the area bounded by $y = 1 - x^2$, and $y = 4 - 4x^2$ about

(i) $x-$axis (ii) the line $y = -1$. [14 marks]

QUESTION 7

(a) Find the length of the curve given by the parametric equations, $x = \cos t, \quad y = t + \sin t, \quad 0 \leq t \leq \pi.$ [6 marks]

(b) (i) Derive the formula of the area of the surface swept out by revolving the graph of a nonegative continuous function $y = f(x) \quad a \leq x \leq b$, about the x-axis. [9 marks]

(ii) Use the formula above to find the area of the surface generated by revolving the curve $y = 2\sqrt{x}$,

$1 \leq x \leq 2$, about the x-axis. [5 marks]

END OF EXAMINATION