UNIVERSITY OF SWAZILAND

FINAL EXAMINATIONS 2008/2009

BSc. / BEd. / B.A.S.S. II

TITLE OF PAPER	:	CALCULUS II
COURSE NUMBER	:	M 212
TIME ALLOWED	:	THREE (3) HOURS
INSTRUCTIONS	:	1. THIS PAPER CONSISTS OF SEVEN QUESTIONS. 2. ANSWER ANY FIVE QUESTIONS 3. ONLY NON-PROGRAMMABLE CALCULATORS MAY BE USED.
SPECIAL REQUIREMENTS	:	NONE

THIS EXAMINATION PAPER SHOULD NOT BE OPENED UNTIL PERMISSION HAS BEEN GRANTED BY THE INVIGILATOR.

QUESTION 1

(a) Replace the polar equation $r^2 = -4r\cos\theta$ by the equivalent cartesian equation. Then describe or identify the graph. [5]

(b) Change $(r,\theta) = (2, \frac{3\pi}{2})$ from polar to rectangular coordinates. [3]

(c) Change $(x,y) = (-1,-1)$ from rectangular to polar coordinates. [4]

(d) Determine the length of $r = \theta, 0 \le \theta \le 1$. [8]

QUESTION 2

(a) Find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$ at $(2,1,-1)$ given that z is a differentiable function of x and y for $\ln(2x^2 + y - z^3) = z$. [10]

(b) Let $T = x^2y - xy^3 + 2$, $x = r\cos\theta$, $y = r\sin\theta$.
Find $\frac{\partial T}{\partial r}$ and $\frac{\partial T}{\partial \theta}$ by the chain rule. [10]

QUESTION 3

(a) Find the area of the region that lies inside the circle $r = 1$ and outside the cardiod $r = 1 - \cos\theta$. [10]

(b) Find the limit and discuss the continuity of the function

$$\lim_{(x,y)\to(1,2)} \frac{x}{\sqrt{2x+y}}.$$

[5]

(c) Verify that $W_{xy} = W_{yx}$ for $W = x\sin y + y\sin x + xy$. [5]

QUESTION 4

(a) Find the volume of the solid whose base is a triangle lying in the xy plane, bounded by the $x-$ axis, the line $x = 1$, and whose top lies in the plane $z = x + y + 1$. [10]

(b) Evaluate the following integral over D, where D is the region bounded by $y = \sqrt{x}$ and $y = x^3$.

$$\iint_D (4xy - y^3)dA.$$

[10]

QUESTION 5

(a) Use polar coordinates to evaluate the following integral over R, where R is the region in the upper half-plane bounded by the circles $x^2 + y^2 = 1$ and $x^2 + y^2 = 4$,

$$\iint_R (3x + 4y^2)dA.$$

[10]

(b) Find the volume of the solid bounded by the paraboloid $4x^2 + 4y^2 + z = 1$ and the $xy-$plane. [10]

QUESTION 6

(a) Use spherical coordinates to evaluate the triple integral;

$$\int_{-1}^{1}\int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}}\int_{-\sqrt{1-x^2-y^2}}^{\sqrt{1-x^2-y^2}}\sqrt{x^2+y^2+z^2}dzdydx.$$

[10]

(b) Express the equation of the cone $z=\sqrt{x^2+y^2}$ in spherical coordinates. [5]

(c) Evaluate

$$\int_0^{\frac{\pi}{2}}\int_1^3 r^2\cos^2\theta drd\theta.$$

[5]

QUESTION 7

(a) Define continuity of a function $f(x,y)$ at a point (x_0,y_0). [6]

(b) State and use the limit definition to show that

$$\lim_{(x,y)\to(x_0,y_0)} x = x_0.$$

[10]

(c) Evaluate

$$\lim_{(x,y)\to(0,\ln 2)} e^{x-y}.$$

[4]